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Quasi-neutral limit of the full Navier–Stokes–Fourier–Poisson system

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Abstract

The quasi-neutral limit of the full Navier–Stokes–Fourier–Poisson system in the torus $\mathbb{T}^d$ $(d \geq 1)$ is considered. We rigorously prove that as the scaled Debye length goes to zero, the global-in-time weak solutions of the full Navier–Stokes–Fourier–Poisson system converge to the strong solution of the incompressible Navier–Stokes equations as long as the latter exists. In particular, the effect of large temperature variations is taken into account.

Keywords: Quasi-neutral limit; Full Navier–Stokes–Fourier–Poisson system; Relative entropy

1. Introduction

In the present paper, we shall consider the singular limit of the following full *Navier–Stokes–Fourier–Poisson* (NSFP) system

$$\partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0, \tag{1.1}$$

$$\partial_t (\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p(\rho, \theta) = \operatorname{div} \mathbb{S}(\theta, \nabla \mathbf{u}) - \rho \nabla \Phi, \tag{1.2}$$

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$$\partial_t\big(\rho s(\rho,\theta)\big)+\operatorname{div}\big(\rho s(\rho,\theta)\mathbf{u}\big)=-\operatorname{div}\left(\frac{\mathbf{q}(\theta,\nabla\theta)}{\theta}\right)+\sigma, \tag{1.3}$$

$$-\lambda^2\Delta\Phi=\rho-1, \tag{1.4}$$

for $x\in\Omega$, $t>0$. Here Ω is the torus $\mathbb{T}^d$, $d\geq 1$. $\lambda>0$ is the (scaled) Debye length. ρ denotes the electron density, $\mathbf{u}$ the velocity, θ the absolute temperature, and Φ the electrostatic potential, respectively. p is the pressure, s is the (specific) entropy and σ stands for the entropy production rate. System (1.1)–(1.4) is a full non-isentropic model involving dissipation and heat diffusion which describes the dynamics of a plasma, where the compressible electron fluid interacts with its own electric field against a constant charged ion background.

Furthermore we assume that the viscous stress $\mathbb{S}$ obeys *Newton's rheological law*:

$$\mathbb{S}(\theta,\nabla\mathbf{u})=\mu(\theta)\left(\nabla\mathbf{u}+\nabla^t\mathbf{u}-\frac{2}{3}\operatorname{div}\mathbf{u}\mathbb{I}\right). \tag{1.5}$$

The heat flux $\mathbf{q}$ satisfies the *Fourier law*:

$$\mathbf{q}(\theta,\nabla\theta)=-\kappa(\theta)\nabla\theta. \tag{1.6}$$

We shall define a concept of weak solution to the NSFP system (1.1)–(1.6) in the same way as in [8,10]. The weak solutions satisfy the system (1.1)–(1.4) in the sense of distributions, and the entropy production rate σ is a non-negative measure satisfying

$$\sigma\geq\frac{1}{\theta}\left(\mathbb{S}(\theta,\nabla\mathbf{u}):\nabla\mathbf{u}-\frac{\mathbf{q}(\theta,\nabla\theta)\cdot\nabla\theta}{\theta}\right). \tag{1.7}$$

In addition, we supplement the system (1.1)–(1.4) with the total energy balance,

$$\frac{d}{dt}\int\left(\frac{1}{2}\rho|\mathbf{u}|^2+\rho e(\rho,\theta)+\frac{1}{2}\lambda^2|\nabla\Phi|^2\right)dx=0, \tag{1.8}$$

where $e=e(\rho,\theta)$ is the (specific) internal energy.

Formally, if we set $\lambda=0$ in the Poisson equation (1.4), we have $\rho=1$, which corresponds to the quasi-neutrality regime in plasma physics. In such situations, the behavior of the fluid can be described by the incompressible Navier–Stokes system. Setting

$$\lambda\Phi\to\phi^\lambda, \tag{1.9}$$

and

$$\mu(\theta)\to\mu\big(\theta^\lambda\big),\qquad \kappa(\theta)\to\kappa\big(\theta^\lambda\big), \tag{1.10}$$

the NSFP system (1.1)–(1.4) can be rewritten as