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J. Differential Equations 258 (2015) 3851–3879

Journal of Differential Equations

www.elsevier.com/locate/jde

Equivalence of linear stabilities of elliptic triangle solutions of the planar charged and classical three-body problems

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Received 11 February 2014; revised 19 January 2015

Available online 14 February 2015

Abstract

In this paper, we prove that the linearized system of elliptic triangle homographic solution of planar charged three-body problem can be transformed to that of the elliptic equilateral triangle solution of the planar classical three-body problem. Consequently, the results of Martínez, Samà and Simó (2006) [15] and results of Hu, Long and Sun (2014) [6] can be applied to these solutions of the charged three-body problem to get their linear stability.

MSC: 70F07; 70H14; 37J45

Keywords: Charged three-body problem; Linear stability; Equivalence; Elliptic relative equilibria

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1 Partially supported by NSFC (No. 11425105) of China.

2 Partially supported by NSFC (No. 11131004), MCME, RFDP, LPMC of the Ministry of Education of China, Nankai University, and the Beijing Center for Mathematics and Information Interdisciplinary Sciences at the Capital Normal University.

http://dx.doi.org/10.1016/j.jde.2015.01.045

1. Main results

We consider the charged planar three-body problem concerns of 3 point particles endowed with a positive mass $m_j \in \mathbf{R}^+ = \{r \in \mathbf{R} \mid r > 0\}$ and an electrostatic charge of any sign $e_j \in \mathbf{R}$, $j = 1, 2, 3$, moving under the influence of the respective Newtonian and Coulombian force. Denote by $q_1, q_2, q_3 \in \mathbf{R}^2$ the position vectors of the three particles respectively. By Newton's second law, the law of universal gravitation and Coulombian's law, the system of equations for this problem is

$$m_i \ddot{q}_i = \sum_{j \neq i} \frac{m_i m_j - e_i e_j}{|q_i - q_j|^3}(q_j - q_i) = \frac{\partial U(q)}{\partial q_i}, \qquad \text{for} \quad i = 1, 2, 3, \tag{1.1}$$

where $U(q) = U(q_1, q_2, q_3) = \sum_{1 \le i < j \le 3} \frac{m_i m_j - e_i e_j}{|q_i - q_j|}$ is the potential or force function by using the standard norm $|\cdot|$ of vectors in $\mathbf{R}^2$. For periodic solutions with period 2π, the system (1.1) is the Euler–Lagrange equation of the action functional

$$\mathcal{A}(q) = \int_0^{2\pi} \left[\sum_{i=1}^{3} \frac{m_i |\dot{q}_i(t)|^2}{2} + U(q(t)) \right] dt$$

defined on the loop space $W^{1,2}(\mathbf{R}/(2\pi\mathbf{Z}), \hat{\mathcal{X}})$, where

$$\hat{\mathcal{X}} = \left\{ q = (q_1, q_2, q_3) \in (\mathbf{R}^2)^3 \;\middle|\; \sum_{i=1}^{3} m_i q_i = 0,\ q_i \neq q_j,\ \forall i \neq j \right\}$$

is the configuration space of the planar three-body problem. Periodic solutions of (1.1) correspond to critical points of the action functional $\mathcal{A}$.

It is a well-known fact that (1.1) can be reformulated as a Hamiltonian system. Let $p_1, p_2, p_3 \in \mathbf{R}^2$ be the momentum vectors of the particles respectively. The Hamiltonian system corresponding to (1.1) is

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}, \ \dot{q}_i = \frac{\partial H}{\partial p_i}, \qquad \text{for} \quad i = 1, 2, 3, \tag{1.2}$$

with Hamiltonian function

$$H(p, q) = H(p_1, p_2, p_3, q_1, q_2, q_3) = \sum_{i=1}^{3} \frac{|p_i|^2}{2m_i} - U(q_1, q_2, q_3). \tag{1.3}$$

Note that if all charges are zero, the problem reduces to the classical Newtonian one. The charged problem has a more complicated dynamical behavior.

Central configurations are basic topics which help understanding the complexity of the charged problem. It is well known that, in the classical Newtonian three-body problem, there are five central configurations: two of them are equilateral triangles and three of them are collinear. In the charged problem, Pérez-Chavela, Saari, Susin and Yan ([17], 1996) proved that there