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Asymptotic stability of thermoelastic systems of Bresse type

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Abstract

We provide a comprehensive stability analysis of the thermoelastic Bresse system (also known as the circular arch problem). In particular, assuming a temperature evolution of Gurtin–Pipkin type, we establish a necessary and sufficient condition for exponential stability in terms of the structural parameters of the problem. As a byproduct, a complete characterization of the longtime behavior of Bresse-type systems with Fourier, Maxwell–Cattaneo and Coleman–Gurtin thermal laws is obtained. Our main theorem also subsumes some recent achievements in the stability properties of thermoelastic Timoshenko systems with classical and nonclassical heat conduction.

MSC: 35B40; 45K05; 47D03; 74D05; 74F05

Keywords: Bresse system; Gurtin–Pipkin law; Contraction semigroup; Exponential stability; Stability number

1. Introduction

Given a real interval $\mathfrak{I} = [0, L]$, we consider the thermoelastic Bresse system with Gurtin–Pipkin thermal dissipation

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$$\begin{cases} \rho_1 \varphi_{tt} - k(\varphi_x + \psi + lw)_x - k_0 l(w_x - l\varphi) = 0, \\ \rho_2 \psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + lw) + \gamma \theta_x = 0, \\ \rho_1 w_{tt} - k_0(w_x - l\varphi)_x + kl(\varphi_x + \psi + lw) = 0, \\ \rho_3 \theta_t - k_1 \int_0^\infty g(s)\theta_{xx}(t-s)\,\mathrm{d}s + \gamma \psi_{tx} = 0, \end{cases} \tag{1.1}$$

in the unknowns variables

$$\varphi, \psi, w, \theta : (x,t) \in \mathfrak{I} \times [0,\infty) \mapsto \mathbb{R}.$$

Here, ρ_1, ρ_2, ρ_3 as well as $b, l, \gamma, k, k_0, k_1$ are strictly positive fixed constants, while g is a bounded convex summable function on $[0,\infty)$ of total mass

$$\int_0^\infty g(s)\,\mathrm{d}s = 1$$

having the explicit form

$$g(s) = \int_s^\infty \mu(r)\,\mathrm{d}r,$$

where $\mu : \mathbb{R}^+ = (0,\infty) \to [0,\infty)$, the so-called memory kernel, is a nonincreasing absolutely continuous function such that

$$\mu(0) = \lim_{s \to 0} \mu(s) \in (0,\infty).$$

In particular, μ is summable on $\mathbb{R}^+$ with

$$\int_0^\infty \mu(s)\,\mathrm{d}s = g(0),$$

and the requirement that g has total mass 1 translates into

$$\int_0^\infty s\mu(s)\,\mathrm{d}s = 1.$$

Moreover, the kernel μ is supposed to satisfy the additional assumption

$$\mu'(s) + \nu\mu(s) \leq 0 \tag{1.2}$$