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Persistence versus extinction under a climate change in mixed environments

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Abstract

This paper is devoted to the study of the persistence versus extinction of species in the reaction–diffusion equation:

$$u_t - \Delta u = f(t, x_1 - ct, y, u) \qquad t > 0, \; x \in \Omega,$$

where Ω is of cylindrical type or partially periodic domain, f is of Fisher-KPP type and the scalar $c > 0$ is a given forced speed. This type of equation originally comes from a model in population dynamics (see [3,17,18]) to study the impact of climate change on the persistence versus extinction of species. From these works, we know that the dynamics is governed by the traveling fronts $u(t, x_1, y) = U(x_1 - ct, y)$, thus characterizing the set of traveling fronts plays a major role. In this paper, we first consider a more general model than the model of [3] in higher dimensional space, where the environment is only assumed to be globally unfavorable with favorable pockets extending to infinity. We consider in two frameworks: the reaction term is time-independent or time-periodic dependent. For the latter, we study the concentration of the species when the environment outside Ω becomes extremely unfavorable and further prove a symmetry breaking property of the fronts.

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Keywords: KPP equations; Traveling wave solutions; Eigenvalue problems; Unfavorable; Concentration; Cylindrical domains

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1. Introduction and main results

1.1. Introduction and definitions

In a pioneering paper [3], Berestycki et al. studied the influence of climate change (global warming) on the population dynamics of biological species, who are strongly sensitive to temperature conditions. The authors proposed a mathematical model in $\mathbb{R}$, which is formulated as a reaction–diffusion equation with a forced speed c:

$$u_t - u_{xx} = f(x - ct, u) \qquad x \in \mathbb{R}, \tag{1.1}$$

where u denotes population density of species and c is the speed of the climate change. A typical f considered in [3] is

$$f(x, s) = \begin{cases} -sm & \text{for } x < 0 \text{ and } x > L \\ sm'\left(1 - \dfrac{s}{K}\right) & \text{for } 0 \le x \le L, \end{cases} \tag{1.2}$$

for some positive constants m, m', L, K. This nonlinearity expresses that the environment is unfavorable outside a compact set $[0, L]$ and favorable inside. The higher dimensional versions with more general type of f were studied later in [7,8]. Beside that a similar model was also considered in the context of competing species by Potapov and Lewis [17], where the authors investigated the co-existence of two species under the effect of climate change and moving range