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Asymptotic behavior for abstract evolution differential equations of second order

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Abstract

Abstract evolution differential equations of second order in time are studied in order to get (almost) optimal decay estimates to the corresponding energy functional of the equations. The framework is supported by a special energy method in the associated Fourier space. The constructed abstract theory can be applied to several concrete evolutionary partial differential equations as is illustrated in the last section of the paper.

MSC: primary 35L90, 35B40; secondary 35E05, 35L15

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1. Introduction

In this work we study decay estimates for the energy and L^2-norm of solutions for the following abstract second order evolution equation

$$A_1 u_{tt}(t,x) + A_2 u_t(t,x) + A_3 u(t,x) = 0 \tag{1.1}$$

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with initial conditions

$$u(0,x) = u_0(x) \quad \text{and} \quad u_t(0,x) = u_1(x)\,, \tag{1.2}$$

where $t \in \mathbb{R}^+$, $x \in \mathbb{R}^n$ and A_i $(i = 1, 2, 3)$ are positive self-adjoint pseudo differential operators with symbols given by functions $P_i(\xi)$ $(i = 1, 2, 3)$. The conditions imposed on these symbols will be described in the next section.

In order to obtain estimates for the problem (1.1)–(1.2) we work with the corresponding problem in the Fourier space. To this end we take the Fourier transform of the problem (1.1)–(1.2) to obtain the following one:

$$\begin{aligned} &P_1(\xi)\,\hat{u}_{tt}(t,\xi) + P_2(\xi)\,\hat{u}_t(t,\xi) + P_3(\xi)\,\hat{u}(t,\xi) = 0\,, \\ &\hat{u}(0,\xi) = \hat{u}_0(\xi) \quad \text{and} \quad \hat{u}_t(0,\xi) = \hat{u}_1(\xi)\,. \end{aligned} \tag{1.3}$$

The above problem becomes an initial value one for a linear ordinary differential equation of the second order with coefficients depending on a frequency parameter $\xi \in \mathbb{R}^n_\xi$. To obtain estimates for the energy and L^2-norm of the solution of (1.3) it is important in the most cases to work separately in the regions of low and high frequencies. Thus, in the next section we work to get several estimates for an abstract initial value problem associated with a linear ordinary differential equation of the second order with coefficients depending on a parameter $\xi \in \Omega$, with Ω a region of $\mathbb{R}^n_\xi$. Then, the decay rates of the L^2-norm and of the energy associated with the problem (1.1)–(1.2) will be a consequence of the results obtained in Section 2. The results depending on the symbols of the operators in equation (1.1). In particular, we can apply such results to get several estimates for the initial value problems associated with the wave, plate, IBq type equations and more.

In Section 2 we construct effective integral inequalities in terms of energy functionals related with the abstract ordinary differential equations (1.3) with variable coefficients depending on the frequency parameter and in Section 3 several applications to the evolutionary partial differential equations with constant coefficients will be introduced. We observe that in all applications of Section 3 it is possible to get decay estimates for the total energy and the L^2-norm of solutions. However, in order not to make this paper too long we only get some estimates for the L^2-norm in the applications in Subsection 3.1. Furthermore, we observe that one of our main contributions lies in introducing a simple new method in abstract form to derive (almost) optimal decay estimates of the energy and several norms of solutions to the abstract second order evolution equations (1.1), which can be applied to get decay estimates for concrete initial value problems associated with partial differential equations from mathematics and physics. For example, initial value problems associated with the wave, respectively, the plate equation are particular cases of these applications. Moreover, the results obtained in Section 3 are new and, in particular, the decay rates obtained in Subsection 3.3 for the total energy of the IBq equation with fractional damping $(-\Delta)^\theta u_t$ $(0 \le \theta \le 1)$ have a novelty for the case $0 < \theta < 1$. The cases $\theta = 0$ and $\theta = 1$ are treated by Wang–Xu in [22] and [21], respectively. We also observe that in the applications given in Section 3 one can deal with more general operators than the Laplace operator, and due to this fact the decay rates depend on the order of such operators.

Let us mention some related works. In the case when $A_1 = A_2 = I$ (I = identity operator), and $A_3 = -\Delta$, which corresponds to the usual damped wave equations, nowadays we have so many related articles even if one restricts only to decay estimates of solutions in some norms and