Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 259 (2015) 5168–5191
Journal of Differential Equations
www.elsevier.com/locate/jde

Asymptotic profile of a parabolic–hyperbolic system with boundary effect arising from tumor angiogenesis

Ming Mei [a,b], Hongyun Peng [c], Zhi-An Wang [d,*]

[a] Department of Mathematics, Champlain College Saint-Lambert, Quebec, J4P 3P2, Canada
[b] Department of Mathematics and Statistics, McGill University, Montreal, Quebec, H3A 2K6, Canada
[c] School of Mathematics, South China University of Technology, Guangzhou 510641, China
[d] Department of Applied Mathematics, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

Received 11 February 2015; revised 12 June 2015

Available online 26 June 2015

Abstract

This paper concerns a parabolic–hyperbolic system on the half space $\mathbb{R}_+$ with boundary effect. The system is derived from a singular chemotaxis model describing the initiation of tumor angiogenesis. We show that the solution of the system subject to appropriate boundary conditions converges to a traveling wave profile as time tends to infinity if the initial data is a small perturbation around the wave which is shifted far away from the boundary but its amplitude can be arbitrarily large.

MSC: 35A01; 35B40; 35B44; 35K57; 35Q92; 92C17

Keywords: Chemotaxis; Traveling wave solutions; Asymptotic stability; Boundary effect; Energy estimates

1. Introduction

To model the dynamics and interaction between signaling molecules vascular endothelial growth factor (VEGF) and vascular endothelial cells during the initiation of tumor angiogenesis, the following PDE–ODE hybrid model was proposed in [12]

* Corresponding author.
E-mail addresses: ming.mei@mcgill.ca (M. Mei), penghy010@163.com (H. Peng), mawza@polyu.edu.hk (Z.-A. Wang).

http://dx.doi.org/10.1016/j.jde.2015.06.022

$$\begin{cases} u_t = (Du_x - \xi u(\ln c)_x)_x, \\ c_t = -\mu uc, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $c(x,t)$ denote the density of vascular endothelial cells and concentration of VEGF, respectively. The parameter $D > 0$ is the diffusivity of endothelial cells, $\xi > 0$ is referred to as the chemotactic coefficient measuring the intensity of chemotaxis and μ denotes the degradation rate of the chemical c. Here the chemical diffusion is neglected since it is far less important than its interaction with endothelial cells as treated in [12].

The striking feature of model (1.1) is that the first equation contains a logarithmic sensitivity function $\ln c$ which is singular at $c = 0$. This singular logarithmic sensitivity was first used by Keller and Segel in their original seminal paper [10] to describe the propagation of traveling wave band formed by bacterial chemotaxis observed in the experiment of Adler [1]. Its mathematical derivation was later given in [28] and biological relevance was provided in [9] by both experimental measurements and model simulations. Therefore the logarithm is a meaningful chemotactic sensitivity representation though it causes great challenges in its mathematical analysis and numerical computations. Hence among other things, the foremost mathematical question is how to resolve the logarithmic singularity in order to being able to carry the analysis forward. Toward this end, a Cole–Hopf type transformation as follows was used in [11,32]

$$v = -\frac{1}{\mu}(\ln c)_x = -\frac{1}{\mu}\frac{c_x}{c} \tag{1.2}$$

which transforms the system (1.1) into a parabolic–hyperbolic system:

$$\begin{cases} u_t - \chi(uv)_x = Du_{xx}, \\ v_t - u_x = 0, \end{cases} \tag{1.3}$$

where $\chi = \mu\xi > 0$. Apparently the transformed system (1.3) is much more manipulable mathematically than the original singular system (1.1) since the singularity vanishes. Therefore the Cole–Hopf transformation (1.2) is the key to open a door to study the singular system (1.1). On the other hand, as a newly derived system of conservation laws from biology, the system (1.3) itself is of great interest to study. There has been an amount of interesting works carried out for the transformed system (1.3). In the one dimensional whole space $\mathbb{R}$, the existence of traveling wavefront solutions of (1.3) was obtained first in [32] and nonlinear stability of traveling wave solutions with large wave amplitude was subsequently established by the third author with his collaborators in a series of works [8,18,19]. The stability of composite waves of (1.3) in $\mathbb{R}$ was proved in [16]. For the bounded domain, there are a few results obtained in [17,31,33] which showed that the asymptotic profile of solutions of (1.3) is a constant in one- and multidimensions if zero-flux boundary conditions are imposed. However it is still unknown how to prescribe the suitable boundary conditions to obtain a non-constant asymptotic profile (such as wave-like solution) for the model (1.3). In this paper, we shall make a step forward to this question by considering the asymptotic behavior of solutions of initial–boundary problem (1.3) in the half-space $\mathbb{R}_+ = [0, \infty)$ with the following initial data

$$(u, v)(x, 0) = (u_0, v_0)(x), \quad x \in \mathbb{R}_+ \tag{1.4}$$

and boundary conditions:

$$u(0,t) = u_-, \quad (u, v)(\infty, t) = (u_+, v_+), \quad t \in \mathbb{R}_+, \tag{1.5}$$