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J. Differential Equations 259 (2015) 5220–5270

Journal of Differential Equations

www.elsevier.com/locate/jde

On parametric Gevrey asymptotics for some nonlinear initial value Cauchy problems

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Received 10 March 2014; revised 9 January 2015

Available online 8 July 2015

Abstract

We study a nonlinear initial value Cauchy problem depending upon a complex perturbation parameter ϵ with vanishing initial data at complex time $t = 0$ and whose coefficients depend analytically on (ϵ, t) near the origin in $\mathbb{C}^2$ and are bounded holomorphic on some horizontal strip in $\mathbb{C}$ w.r.t. the space variable. This problem is assumed to be non-Kowalevskian in time t, therefore analytic solutions at $t = 0$ cannot be expected in general. Nevertheless, we are able to construct a family of actual holomorphic solutions defined on a common bounded open sector with vertex at 0 in time and on the given strip above in space, when the complex parameter ϵ belongs to a suitably chosen set of open bounded sectors whose union form a covering of some neighborhood Ω of 0 in $\mathbb{C}^*$. These solutions are achieved by means of Laplace and Fourier inverse transforms of some common ϵ-depending function on $\mathbb{C} \times \mathbb{R}$, analytic near the origin and with exponential growth on some unbounded sectors with appropriate bisecting directions in the first variable and exponential decay in the second, when the perturbation parameter belongs to Ω. Moreover, these solutions satisfy the remarkable property that the difference between any two of them is exponentially flat for some integer order w.r.t. ϵ. With the help of the classical Ramis–Sibuya theorem, we obtain the existence of a formal series (generally divergent) in ϵ which is the common Gevrey asymptotic expansion of the built up actual solutions considered above.

MSC: 35C10; 35C20

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1 The author is partially supported by the project MTM2012-31439 of Ministerio de Ciencia e Innovación, Spain.

http://dx.doi.org/10.1016/j.jde.2015.06.020
0022-0396/

Keywords: Asymptotic expansion; Borel–Laplace transform; Fourier transform; Nonlinear integro-differential equation; Nonlinear partial differential equation; Singular perturbation

1. Introduction

In this paper, we consider a family of parameter depending nonlinear initial value Cauchy problems of the form

$$Q(\partial_z)(\partial_t u(t,z,\epsilon)) = (Q_1(\partial_z)u(t,z,\epsilon))(Q_2(\partial_z)u(t,z,\epsilon)) + \sum_{l=1}^{D} \epsilon^{\Delta_l} t^{d_l} \partial_t^{\delta_l} R_l(\partial_z)u(t,z,\epsilon) \\ + c_0(t,z,\epsilon)R_0(\partial_z)u(t,z,\epsilon) + f(t,z,\epsilon) \quad (1)$$

for given vanishing initial data $u(0,z,\epsilon) \equiv 0$, where $D \geq 2$, Δ_l, d_l, δ_l, $1 \leq l \leq D$ are integers which satisfy the inequalities

$$1 = \delta_1, \quad \delta_l < \delta_{l+1}, \quad d_D = (\delta_D - 1)(k+1), \quad \Delta_D = d_D - \delta_D + 1, \\ d_l > (\delta_l - 1)(k+1), \quad \delta_D \geq \delta_l + \frac{2}{k}, \quad \Delta_l + k(1-\delta_D) + 1 \geq 0$$

for all $1 \leq l \leq D-1$ and for some integer $k \geq 1$. Besides, $Q(X)$, $Q_1(X)$, $Q_2(X)$, $R_l(X)$, $0 \leq l \leq D$ are polynomials submitted to the constraints

$$\deg(Q) \geq \deg(R_D) \geq \deg(R_l), \quad \deg(R_D) \geq \deg(Q_1), \quad \deg(R_D) \geq \deg(Q_2), \\ Q(im) \neq 0, \quad R_D(im) \neq 0$$

for all $m \in \mathbb{R}$, all $0 \leq l \leq D-1$. The coefficient $c_0(t,z,\epsilon)$ and the forcing term $f(t,z,\epsilon)$ are bounded holomorphic functions on a product $D(0,r) \times H_\beta \times D(0,\epsilon_0)$, where $D(0,r)$ (resp. $D(0,\epsilon_0)$) is a disc centered at 0 with small radius $r > 0$ (resp. $\epsilon_0 > 0$) and $H_\beta = \{z \in \mathbb{C}/|\mathrm{Im}(z)| < \beta\}$ is some strip of width $\beta > 0$. In order to avoid cumbersome statements and to improve the readability of the computations, we have restricted our study to a quadratic nonlinearity and monomial coefficients in t in front of the derivatives with respect to t and z but the method described here can also be extended to higher order nonlinearities, with polynomial coefficients w.r.t. t in the linear part on the right handside of equation (1).

This work can be seen as a continuation of the study described in [22] where the second author has studied nonlinear integro-differential initial values problems with the shape

$$R(\partial_z)P(\partial_t,\partial_z)Y(t,z) = \int_0^t b(t-s,z)\partial_z^{s_0}Y(s,z)ds + \int_0^t \partial_z^{s_1}Y(t-s,z)\partial_z^{s_2}Y(s,z)ds \quad (2)$$

where $R(X) \in \mathbb{C}[X]$, $P(T,X) \in \mathbb{C}[T,X]$ and $s_0, s_1, s_2 \geq 0$ are non negative integers. The coefficient $b(t,z) = \sum_{k \in I} b_k(z)t^k$ is a polynomial in t and its coefficients $b_k(z)$ are the Fourier inverse transform of some function $\mathfrak{b}_k(m)$ belonging to a Banach space $E_{(\beta,\mu)}$ of continuous functions