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On a nonlocal reaction–diffusion–advection system modelling the growth of phytoplankton with cell quota structure

Sze-Bi Hsu [a,1], Linfeng Mei [b,a,*,2], Feng-Bin Wang [c,1]

[a] *Department of Mathematics and The National Center for Theoretical Sciences, National Tsing Hua University, Hsinchu 300, Taiwan*

[b] *Department of Mathematics, Henan Normal University, Xinxiang 453007, PR China*

[c] *Department of Natural Science in the Center for General Education, Chang Gung University, Kwei-Shan, Taoyuan 333, Taiwan*

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Abstract

Phytoplankton species in a water column compete for mineral nutrients and light, and the existing models usually neglect differences in the nutrient content and the amount of light absorbed of individuals. In this current paper, we examine a size-structured and nonlocal reaction–diffusion–advection system which describes the dynamics of a single phytoplankton species in a water column where the species depends simply on light for its growth. Our model is under the assumption that the amount of light absorbed by individuals is proportional to cell size, which varies for populations that reproduce by simple division into two equally-sized daughters. We first establish the existence of a critical death rate and our analysis indicates that the phytoplankton survives if and only if its death rate is less than the critical death rate. The critical death rate depends on a general reproductive rate, the characteristics of the water column (e.g., turbulent diffusion rate, sinking, depth), cell growth, cell division, and cell size.

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* Corresponding author.
E-mail address: lfmei@outlook.com (L. Mei).

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1. Introduction

Classical phytoplankton competition studies often assume a simple, well-mixed laboratory system, such as the chemostat culture system, in which a nutrient medium is pumped, balanced by an outflow that removes nutrients and organisms [30]. The chemostat is a basic piece of ideal apparatus and it has been thought of as a lake or pond in a laboratory. However, in many aquatic environments, the habitat may be poorly mixed, and have spatial gradients of resource availability. For example, the vertical transport (motion) of phytoplankton species in the water column is determined by vertical turbulent diffusion and advection (sinking or buoyant).

Another factor that may affect the competition between species for resources is size structure of algal communities. The simplest competition models neglect differences between individuals, assuming a constant quota of resource per individual [12,17]. In fact, quotas may vary. The variable-internal-stores models [11,18,29] assume that all individuals have the same quota at any instant and the dynamics of quota for a species is governed by an ordinary differential equation. Alternatively, Diekmann et al. [3,7,16] proposed a structured population model in which quotas may differ among individuals at any instant.

Investigation of the mechanisms contributing to the emergence of size structures in spatially varying environments is a challenging issue in mathematical ecology. Resource storage within individuals leads to population structure and it must be combined with the spatial variation of the environment. There are at least three possible ways to this issue. One is the Lagrangian modelling approach [13] which assumes that each competitor population is divided into many subpopulations that move through two model habitats with gradient in nutrient availability. This model can not be analyzed mathematically and require extensive computation to achieve results. A second approach is an approximation that averages over differences among individuals at a given location, in their amounts of stored nutrient. Equivalently, one assumes that at any location, all individuals have the same quota, as if there were instantaneous redistribution of resources among individuals at the same place. This assumption yields a set of analytically and computationally tractable partial differential equations [14,20,23,24]. The third one combines the structured population model proposed in [3,7,16] with the physical transport equations governing spatial distributions of populations and nutrients. In [15], the authors assume nutrient content of individuals is proportional to cell size and the habitat is taken to be an unstirred chemostat where organisms and nutrients move by simple diffusion.

The growth of population depends critically on the supply of two fundamental types of resources: light and mineral nutrients. In phytoplankton communities, species typically compete for nutrient and light which are complementary resources for their growth [4,5,21,25]. There are also two possibly extreme cases. In oligotrophic ecosystems with ample supply of light, they tend to compete only for nutrients [26,28], and in eutrophic environments with ample nutrients supply, they compete only for light [6,8,9,19,22].

In this current paper, we focus on the study of the dynamics of a single species in a water column in eutrophic ecosystem, that is, the species depends only on light for its growth. As in [15], we shall assume the amount of light absorbed by individuals is proportional to cell size, which varies for populations that reproduce by simple division into two equally-sized daughters, and species move by vertical turbulent diffusion and advection (sinking or buoyant). Most of phytoplankton species have tendency to sink as they are heavier than water while some species