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A mathematical and numerical framework for magnetoacoustic tomography with magnetic induction [☆]

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Abstract

We provide a mathematical analysis and a numerical framework for magnetoacoustic tomography with magnetic induction. The imaging problem is to reconstruct the conductivity distribution of biological tissue from measurements of the Lorentz force induced tissue vibration. We begin with reconstructing from the acoustic measurements the divergence of the Lorentz force, which is acting as the source term in the acoustic wave equation. Then we recover the electric current density from the divergence of the Lorentz force. To solve the nonlinear inverse conductivity problem, we introduce an optimal control method for reconstructing the conductivity from the electric current density. We prove its convergence and stability. We also present a point fixed approach and prove its convergence to the true solution. A new direct reconstruction scheme involving a partial differential equation is then proposed based on viscosity-type regularization to a transport equation satisfied by the electric current density field. We prove that solving such an equation yields the true conductivity distribution as the regularization parameter approaches zero. Finally, we test the three schemes numerically in the presence of measurement noise, quantify their stability and resolution, and compare their performance.

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1. Introduction

The Lorentz force plays a key role in magneto-acoustic tomographic techniques [36]. Several approaches have been developed with the aim of providing electrical impedance information at a spatial resolution on the scale of ultrasound wavelengths. These include ultrasonically-induced Lorentz force imaging [16,26] and magneto-acoustic tomography with magnetic induction [43,37].

Electrical conductivity varies widely among soft tissue types and pathological states [23,35] and its measurement can provide information about the physiological and pathological conditions of tissue [14]. Acousto-magnetic tomographic techniques have the potential to detect small conductivity inhomogeneities, enabling them to diagnose pathologies such as cancer by detecting tumorous tissues when other conductivity imaging techniques fail to do so.

In magnetoacoustic imaging with magnetic induction, magnetic fields are used to induce currents in the tissue. Ultrasound is generated by placing the tissue in a dynamic and static magnetic field. The dynamic field induces eddy currents and the static field leads to generation of acoustic vibration from Lorentz force on the induced currents. The divergence of the Lorentz force acts as acoustic source of propagating ultrasound waves that can be sensed by ultrasonic transducers placed around the tissue. The imaging problem is to obtain the conductivity distribution of the tissue from the acoustic source map; see [31–34,44].

This paper provides a mathematical and numerical framework for magnetoacoustic imaging with magnetic induction. We develop efficient methods for reconstructing the conductivity in the medium from the Lorentz force induced vibration. For doing so, we first estimate the electric current density in the tissue. Then we design efficient algorithms for reconstructing the heterogeneous conductivity map from the electric current density with the ultrasonic resolution.

The paper is organized as follows. We start by describing the forward problem. Then we reconstruct from the acoustic measurements the divergence of the Lorentz force, which is acting as the source term in the acoustic wave equation. We recover the electric current density from the divergence of the Lorentz force, which reduces the problem to imaging the conductivity from the internal electric current density. We introduce three reconstruction schemes for solving the conductivity imaging problem from the internal electric current density. The first is an optimal control method. One of the contributions of this paper is the proof of convergence and stability of the optimal control approach provided that two magnetic excitations leading to nonparallel current densities are employed. Then we present a point fixed approach and prove that it converges to the true conductivity image. Finally, we propose an alternative to these iterative schemes via the use of a transport equation satisfied by the internal electric current density. Our third algorithm is direct and can be viewed as a PDE-based reconstruction scheme. We test numerically the three proposed schemes in the presence of measurement noise, and also quantify their stability and resolution.

The feasibility of imaging of Lorentz-force-induced motion in conductive samples was shown in [21]. The magnetoacoustic tomography with magnetic induction investigated here was experimentally tested in [33,34], and was reported to produce conductivity images of quality comparable to that of ultrasound images taken under similar conditions. Other emerging hybrid techniques for conductivity imaging have also been reported in [1–10,12,17,24,38,39,42].

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