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Global existence and optimal decay rates for the Timoshenko system: The case of equal wave speeds

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Abstract

We first show the global existence and optimal decay rates of solutions to the classical Timoshenko system in the framework of Besov spaces. Due to the *non-symmetric* dissipation, the general theory for dissipative hyperbolic systems (see [31]) cannot be applied to the Timoshenko system directly. In the case of equal wave speeds, we construct global solutions to the Cauchy problem pertaining to data in the spatially Besov spaces. Furthermore, the dissipative structure enables us to give a new decay framework which pays less attention on the traditional spectral analysis. Consequently, the optimal decay estimates of solution and its derivatives of fractional order are shown by time-weighted energy approaches in terms of low-frequency and high-frequency decompositions. As a by-product, the usual decay estimate of $L^1(\mathbb{R})$-$L^2(\mathbb{R})$ type is also shown.

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Keywords: Global existence; Optimal decay estimates; Critical Besov spaces; Timoshenko system

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1. Introduction

Consider the following Timoshenko system (see [28,29]), which is a set of two coupled wave equations of the form

$$\begin{cases} \varphi_{tt} - (\varphi_x - \psi)_x = 0, \\ \psi_{tt} - \sigma(\psi_x)_x - (\varphi_x - \psi) + \gamma\psi_t = 0, \end{cases} \tag{1.1}$$

and describes the transverse vibrations of a beam. Here $t \geq 0$ is the time variable, $x \in \mathbb{R}$ is the spacial variable which denotes the point on the center line of the beam, $\varphi(t, x)$ is the transversal displacement of the beam from an equilibrium state, and ψ is the rotation angle of the filament of the beam. The smooth function $\sigma(\eta)$ satisfies $\sigma'(\eta) > 0$ for any $\eta \in \mathbb{R}$, and γ is a positive constant. System (1.1) is supplemented with the initial data

$$(\varphi, \varphi_t, \psi, \psi_t)(x, 0) = (\varphi_0, \varphi_1, \psi_0, \psi_1)(x). \tag{1.2}$$

The linearized system of (1.1) reads correspondingly as

$$\begin{cases} \varphi_{tt} - (\varphi_x - \psi)_x = 0, \\ \psi_{tt} - a^2\psi_{xx} - (\varphi_x - \psi) + \gamma\psi_t = 0, \end{cases} \tag{1.3}$$

with $a > 0$ is the sound speed defined by $a^2 = \sigma'(0)$. The case $a = 1$ corresponds to the Timoshenko system with equal wave speeds.

1.1. Known results

In a bounded domain, it is known that (1.3) is exponentially stable if the damping term φ_t is also present on the left-hand side of the first equation of (1.3) (see, e.g., [21]). Soufyane [27] showed that (1.3) could not be exponentially stable by considering only the damping term of the form ψ_t, unless for the case of $a = 1$ (equal wave speeds). A similar result was obtained by Rivera and Racke [23] with an alternative proof. Moreover, Rivera and Racke [22] extended those results in [21,23] to the Timoshenko system where the heat conduction described by the classical Fourier law was additional considered.

In the whole space, the third author et al. [10] introduced the following quantities

$$v = \varphi_x - \psi, \qquad u = \varphi_t, \qquad z = a\psi_x, \qquad y = \psi_t, \tag{1.4}$$

so that the system (1.3) can be rewritten as

$$\begin{cases} v_t - u_x + y = 0, \\ u_t - v_x = 0, \\ z_t - ay_x = 0, \\ y_t - az_x - v + \gamma y = 0. \end{cases} \tag{1.5}$$

The initial data are given by

$$(v, u, z, y)(x, 0) = (v_0, u_0, z_0, y_0)(x), \tag{1.6}$$