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On the expansion of a wedge of van der Waals gas into a vacuum

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Abstract

We study the existence of global in time classical solution to the expansion of a wedge of van der Waals gas into a vacuum. We reduce this problem to a Goursat-type boundary value problem for 2D self-similar Euler system. The 2D self-similar Euler system is a mixed type system, the type in each point is determined by the local pseudo-Mach number. By introducing the concept of invariant square region of solution, we prove that this system is strictly hyperbolic in the flow region of the Goursat problem. A prior C^1 estimate of the solution to the Goursat problem is obtained by using the method of characteristic decomposition. Due to the existence of vacuum boundary, the classical approach to extend local solution to global solution does not work here. We extend the local solution of the Goursat problem up to the interface of gas with vacuum by solving many "small" Goursat problems in each extension step.

MSC: primary 35L65, 35J70, 35R35; secondary 35J65

Keywords: 2D Riemann problem; Van der Waals gas; Gas expansion; Vacuum; Planar rarefaction wave; Wave interaction

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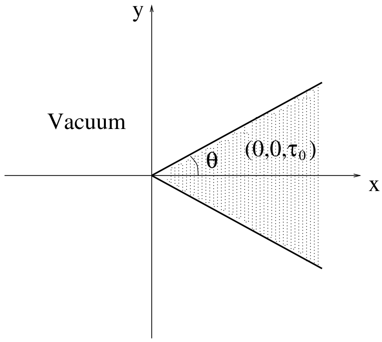

Fig. 1. Initial data of the expansion of a wedge of gas into a vacuum.

1. Introduction

Two-dimensional (2D) Riemann problems, which usually refer to Cauchy problems with initial data that are constant along each ray from the origin, are interesting and important open problems in the field of hyperbolic systems of conservation laws. Recently, there is much progress in 2D Riemann problems for the compressible Euler equations and some related models; see [4,10] for potential flow equation, [7,11,14,15,18,19,30,34] for isentropic compressible Euler equations, [3] for UTSD model, [5,6,12,26] for Chaplygin gas equations, [1,9,13,36] for pressure gradient system, and [27] for transport equations. We also refer the reader to [16,33,35] for rich flow structures of solutions to 2D Riemann problems.

In this paper, we are concerned with the expansion of a wedge of gas into a vacuum. This problem can be formulated as the following 2D Riemann problem for the isentropic compressible Euler equations:

$$\begin{cases} \rho_t + (\rho u)_x + (\rho v)_y = 0, \\ (\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = 0, \\ (\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = 0, \end{cases} \tag{1.1}$$

$$(u, v, \rho)(x, y, 0) = \begin{cases} (0, 0, \rho_0), & (x, y) \in \{x > 0,\ -x\tan\theta < y < x\tan\theta\}; \\ \text{vacuum}, & \text{otherwise}, \end{cases} \tag{1.2}$$

where ρ is the density, (u, v) is the velocity, $p = p(\rho)$ is the pressure, θ is the half-angle of the wedge, and $\rho_0 > 0$ is a constant. See Fig. 1.

This gas expansion problem has been an interesting problem for a long time, it catches some important types of wave interaction. When the gas is a polytropic ideal gas, this problem can be reduced to the interaction of two planar rarefaction waves, and the global in time existence of classical solution has been solved completely; see [7,14,15,18,30,34]. A natural question is to determine whether these results can be extended to more realistic gases, for example, the van der Waals gas.

In this paper, we consider a polytropic van der Waals gas, for which the equation of state is

$$p = \frac{A}{(\tau - b)^{\delta+1}} - \frac{a}{\tau^2}, \tag{1.3}$$