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Entire solutions for nonlocal dispersal equations with spatio-temporal delay: Monostable case

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Abstract

This paper deals with entire solutions for a general nonlocal dispersal monostable equation with spatio-temporal delay, i.e., solutions that are defined in the whole space and for all time $t \in \mathbb{R}$. We first derive a particular model for a single species and show how such systems arise from population biology. Then we construct some new types of entire solutions other than traveling wave solutions and equilibrium solutions of the equation under consideration with quasi-monotone and non-quasi-monotone nonlinearities. Various qualitative properties of the entire solutions are also investigated. In particular, the relationship between the entire solutions and the traveling wave fronts which they originated is considered. Our main arguments are based on the comparison principle, the method of super- and sub-solutions, and the construction of auxiliary control systems.

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Keywords: Entire solution; Traveling wave solution; Nonlocal dispersal equation; Spatio-temporal delay; Monostable nonlinearity

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1. Introduction

In recent years, many reaction–diffusion equations with *spatio-temporal delay or nonlocal delay* have been proposed and studied to model the interactions of time lag of feedback and spatial diffusion of biological species. See the survey papers of Gourley and Wu [13] and Ruan [32]. Two typical and important examples which have been extensively studied are the diffusive Nicholson's blowflies equation with spatio-temporal delay [10,11,20]:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} - \delta u(x,t) + p(G*u)(x,t)\exp\big\{-a(G*u)(x,t)\big\}, \tag{1.1}$$

and the following equation describing the evolution of matured population of a single species [1,12,33]:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} - du(x,t) + \int_0^\infty \int_{-\infty}^{+\infty} G(x-y,s)b\big(u(y,t-s)\big)dyds, \tag{1.2}$$

where $u(x,t)$ denotes the density of the population at location $x \in \mathbb{R}$ and time $t \geq 0$, $D, \delta, p, a, d > 0$ are constants, $G(\cdot,\cdot)$ is the kernel function, and

$$(G*u)(x,t) = \int_0^\infty \int_{-\infty}^{+\infty} G(x-y,s)u(y,t-s)dyds. \tag{1.3}$$

Note that a basic assumption behind (1.1) and (1.2) is that the internal interaction of species is random and local, i.e. individuals move randomly between the adjacent spatial locations. In reality, the movements and interactions of many species in ecology and biology can occur between non-adjacent spatial locations, see e.g. Lee et al. [18] and Murray [29]. Taking this fact into account, (1.1) and (1.2) can be extended to the following *nonlocal dispersal equations with spatio-temporal delay*:

$$\frac{\partial u}{\partial t} = D(J*u-u)(x,t) - \delta u + p(G*u)(x,t)\exp\big\{-a(G*u)(x,t)\big\}, \tag{1.4}$$

and

$$\frac{\partial u}{\partial t} = D(J*u-u)(x,t) - du + \int_0^\infty \int_{-\infty}^{+\infty} G(x-y,s)b\big(u(y,t-s)\big)dyds, \tag{1.5}$$

respectively, where $(G*u)(x,t)$ is defined in (1.3), $J*u-u$ is a nonlocal dispersal operator and $J*u$ is a spatial convolution defined by

$$(J*u)(x,t) = \int_{-\infty}^{+\infty} J(x-y)u(y,t)dy. \tag{1.6}$$