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Stability of transonic jet with strong shock in two-dimensional steady compressible Euler flows

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Abstract

For steady supersonic flow past a solid convex corner surrounded by quiescent gas, if the pressure of the upcoming supersonic flow is lower than the pressure of the quiescent gas, there may appear a strong shock to increase the pressure and then a transonic characteristic discontinuity to separate the supersonic flow behind the shock-front from the still gas. In this paper, we prove the global existence, uniqueness, and stability of such flow patterns under suitable conditions on the upstream supersonic flow and the pressure of the surrounding quiescent gas, for the two-dimensional steady complete compressible Euler system. Mathematically, a global weak solution to a characteristic free boundary problem of hyperbolic conservation laws is constructed and shown to be unique and stable under the framework of front tracking method.

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Keywords: Compressible Euler equations; Transonic; Characteristic discontinuity; Free boundary; Front tracking; Glimm functional

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Contents

1. Introduction

As illustrated in Fig. 1, for a steady supersonic flow passing a solid convex corner surrounded by static gas, if the pressure of the quiescent gas is larger than the pressure of the upcoming supersonic flow, there may appear a shock to increase the pressure of the supersonic flow to that of the quiescent gas and a characteristic discontinuity (also called contact discontinuity in gas dynamics, which is a combination of vortex sheet and/or entropy wave) to separate the supersonic flow behind the shock-front from the static gas. In this paper, we are going to prove the existence and uniqueness as well as stability of such a flow pattern using the two-dimensional steady complete compressible Euler system, under reasonable assumptions on the upstream supersonic flow and the lower downstream quiescent gas.

The Euler system governing the two-dimensional steady compressible flows consists of the following conservation laws of mass, momentum and energy: