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Fredholm transform and local rapid stabilization for a Kuramoto–Sivashinsky equation

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Abstract

This paper is devoted to the study of the local rapid exponential stabilization problem for a controlled Kuramoto–Sivashinsky equation on a bounded interval. We build a feedback control law to force the solution of the closed-loop system to decay exponentially to zero with arbitrarily prescribed decay rates, provided that the initial datum is small enough. Our approach uses a method we introduced for the rapid stabilization of a Korteweg–de Vries equation. It relies on the construction of a suitable integral transform and can be applied to many other equations.

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Keywords: Kuramoto–Sivashinsky equation; Rapid stabilization; Integral transform

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1. Introduction

Consider the following Kuramoto–Sivashinsky equation:

$$\begin{cases} v_t + v_{xxxx} + \lambda v_{xx} + v v_x = 0 & \text{in } (0,1)\times(0,+\infty), \\ v(t,0) = v(t,1) = 0 & \text{on } (0,+\infty), \\ v_{xx}(t,0) = f(t),\ v_{xx}(t,1) = 0 & \text{on } (0,+\infty), \\ v(0,\cdot) = v^0(\cdot) & \text{in } (0,1), \end{cases} \tag{1.1}$$

where $\lambda > 0$ and $v^0(\cdot) \in L^2(0,1)$.

For $T > 0$, let us define

$$X_T \overset{\triangle}{=} C^0([0,T]; L^2(0,1)) \cap L^2(0,T; H^2(0,1) \cap H_0^1(0,1)),$$

which is endowed with the norm

$$|\cdot|_{X_T} = \left(|\cdot|^2_{C^0([0,T];L^2(0,1))} + |\cdot|^2_{L^2(0,T;H^2(0,1)\cap H_0^1(0,1))}\right)^{\frac{1}{2}}.$$

We first present the following locally well-posedness result, which is proved in Appendix A of this paper.

Theorem 1.1. *Let $F : L^2(0,1) \to \mathbb{R}$ be a continuous linear map and let $T_0 \in (0,+\infty)$. Then for given $v^0 \in L^2(0,1)$, there exists at most one solution $v \in X_{T_0}$ of (1.1) with $f(t) = F(v(t,\cdot))$.*

Moreover, there exist $r_0 > 0$ and $C_0 > 0$ such that, for every $v^0 \in L^2(0,1)$ with

$$|v^0|_{L^2(0,1)} \le r_0, \tag{1.2}$$

there exists one solution $v \in X_{T_0}$ of (1.1) with $f(t) = F(v(t,\cdot))$ and this solution satisfies

$$|v|_{X_T} \le C_0 |v_0|_{L^2(0,1)}. \tag{1.3}$$

The Kuramoto–Sivashinsky (K–S for short) equation was first derived in [24] as a model for Belouzov–Zabotinskii reaction patterns in reaction–diffusion systems. It describes a lot of physical and chemical systems, such as unstable flame fronts (see [28] for example), falling liquid films (see [10] for example) and interfacial instabilities between two viscous fluids (see [20] for example). Since the pioneer works in [15,26], the well-posedness and dynamical properties of the K–S equation were well studied (see [7,12,16,17,31] and the references therein).

We are interested in the stabilization problems of the K–S equation. There are many degrees of freedom to choose the feedback controls. For instance, one can choose the internal controls, the Dirichlet boundary control, the Neumann boundary control, etc., and the feedback law can be linear or nonlinear with respect to the output. Such kind of problems were studied extensively in the literature. Let us recall some of them.

In [1,11], the authors studied the global stabilization for the K–S equation with periodic boundary conditions. The feedback controller acts on the whole domain. They first considered the ordinary differential equation approximations of the system, which accurately describe the