Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations 259 (2015) 3955–3975

Journal of Differential Equations

www.elsevier.com/locate/jde

Local analyticity radii of solutions to the 3D Navier–Stokes equations with locally analytic forcing

Zachary Bradshaw *, Zoran Grujić, Igor Kukavica

Received 19 June 2014; revised 21 January 2015

Available online 19 May 2015

Abstract

We introduce a new method for establishing local analyticity and estimating the local analyticity radius of a solutions to the 3D Navier–Stokes equations at interior points. The approach is based on rephrasing the problem in terms of second order parabolic systems which are then estimated using the mild solution approach. The estimates agree with the global analyticity radius from [16] up to a logarithm.

MSC: 35Q30; 76D05

Keywords: Navier–Stokes equations; Strong solutions; Analyticity

1. Introduction

In this paper we consider the analyticity radius of solutions to the 3D Navier–Stokes system

$$\begin{aligned} \partial_t u - \Delta u &= -u \cdot \nabla u - \nabla p + f \quad && \text{in } \mathbb{R}^3 \times (0,T) \\ \nabla \cdot u &= 0 && \text{in } \mathbb{R}^3 \times (0,T), \end{aligned} \tag{3D NSE}$$

where the force f is locally analytic. In the first result we provide a lower bound for the analyticity radius of strong solutions to 3D NSE evolving from initial data in L^q where $q > 3$. In the second result we estimate the analyticity radius of locally smooth solutions from below in terms

* Corresponding author.
E-mail address: zbradshaw@math.ubc.ca (Z. Bradshaw).

http://dx.doi.org/10.1016/j.jde.2015.05.009
0022-0396/

of strictly local quantities including the local Reynolds number. This is formulated in purely local terms and applies to the boundary value Navier–Stokes problem yielding estimates for the local analyticity radius of solutions at interior points.

The main motivation for studying the analyticity radii of solutions to viscous fluid models is their connection to the dissipative length scales from turbulence theories [9,11–13,19,20,32]. At and below the dissipative scale, inertial range cascade dynamics break down and frictional effects become the dominant influence on energy transport dynamics. In analytic solutions this shift is visible as the exponential fall off of the Fourier spectrum at frequencies beyond the inverse of the analyticity radius. Another motivation for studying the analyticity radius reflects its applicability to geometric measure-type regularity criteria (see [6,15]). In particular, the radius of spatial analyticity has been identified as the scale of local, anisotropic diffusion in the vorticity formulation of the 3D NSE.

Classical analyticity results for solutions to 3D NSE can be found in [2,14,23,31]. A pioneering work in the area of estimating analyticity radii was carried out by Foias and Temam in [10] using Fourier techniques and Gevrey spaces in an L^2 setting (see also [8,33]). Related results in L^p spaces were obtained in [27,28]. This approach has subsequently been revisited using more modern techniques in a variety of function spaces (see, e.g., [1,3–5,21,30,33,34]). An alternative approach to the problem in L^p spaces where $p \in (3, \infty]$ was developed in [16,18,24] and is carried out entirely in physical space. This strategy is more tailored to accommodate local settings than the Fourier space techniques, a fact shown in [17] where it is applied to solutions of a non-linear heat equation at interior points of bounded domains. Gevrey space techniques have also been applied to study the decay of analyticity radii associated with solutions to the 3D Euler Equations evolving from analytic initial data on the whole space (see [26]) and domains with boundaries (see [25]).

Most available analyticity results for solutions to 3D NSE are global and require the forcing term f to be analytic with a uniform analyticity radius. In this paper we provide a new method which allows us to estimate the analyticity radius in local contexts using the mild solution approach as in [7,22]. In contrast to the global argument of [16], our strategy involves solving second order parabolic systems. The resulting argument is simple and provides a new approach at interior points of bounded domains which avoids the need for complicated recursive estimates like those found in [23].

The paper is organized as follows. In Section 2 we formulate our local assumptions on the forcing and state the main results, Theorems 2.1 and 2.3. The approximation schemes for our inductive arguments are presented in Section 3 where we also prove Theorem 2.1. Section 4 is dedicated to proving Theorem 2.3.

2. Statement of main results

Our first result provides a local lower bound on the analyticity radius of a flow subjected to real-analytic forcing f possessing a possibly non-uniform analyticity radius. With $\lambda_f(x,t)$ denoting the radius of spatial analyticity of f at (x,t), let

$$\lambda_{f,T}(x) = \inf_{t\in(0,T]} \lambda_f(x,t).$$

For x_* in $\mathbb{R}^3$ fixed, denote by B_* the ball of radius $r_* = \lambda_{f,T}(x_*)/2$ centered at x_*. Then, if $x \in 2B_*$ it follows that $\lambda_{f,T}(x) \geq 2r_* - |x - x_*|$, and, consequently, $f(x,t)$ is the restriction