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Sweeping process by prox-regular sets in Riemannian Hilbert manifolds

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Abstract

In this paper, we deal with sweeping processes on (possibly infinite-dimensional) Riemannian Hilbert manifolds. We extend the useful notions (proximal normal cone, prox-regularity) already defined in the setting of a Hilbert space to the framework of such manifolds. Especially we introduce the concept of local prox-regularity of a closed subset in accordance with the geometrical features of the ambient manifold and we check that this regularity implies a property of hypomonotonicity for the proximal normal cone. Moreover we show that the metric projection onto a locally prox-regular set is single-valued in its neighborhood. Then under some assumptions, we prove the well-posedness of perturbed sweeping processes by locally prox-regular sets.

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1. Introduction

1.1. A brief review of results on sweeping processes

The aim of this paper is to extend the study of so-called *sweeping process* in the setting of a (possibly infinite dimensional) Riemannian manifold.

Sweeping processes are specific differential inclusions of first order involving proximal normal cones to a moving set. They were extensively studied for the last years in the Euclidean space and more generally in a Hilbert space.

More precisely, consider a Hilbert space H and a moving set $t \to C(t)$ on a time-interval $I := [0, T]$ and assume that the set-valued map $C : I \rightrightarrows H$ takes nonempty closed values. A function $u : I \to H$ is a solution of a problem of perturbed sweeping process if it satisfies the following differential inclusion:

$$\begin{cases} \dfrac{du(t)}{dt} + \mathrm{N}(C(t), u(t)) - F(t, u(t)) \ni 0 \\ u(t) \in C(t) \\ u(0) = u_0 \, , \end{cases} \tag{1}$$

with an initial data $u_0 \in C(0)$ and $F : I \times H \rightrightarrows H$ a set-valued map taking nonempty compact values. Here, $\mathrm{N}(C(t), u(t))$ stands for the proximal normal cone to $C(t)$ at the point $u(t)$. This differential inclusion can be thought as following: the point $u(t)$, submitted to the field $F(t, u(t))$, has to live in the set $C(t)$ and so follows its time-evolution.

Let us first give a brief overview of the study for such problems. The sweeping processes have been introduced by J.J. Moreau in 70's (see [22]). He considered the following problem: a point $u(t)$ has to be inside a moving convex set $C(t)$ included in a Hilbert space. When this point is catched-up by the boundary of $C(t)$, it moves in the opposite of the outward normal direction of the boundary, as if it was pushed by the physical boundary in order to stay inside the convex set $C(t)$. Then the position $u(t)$ of this point is described by the following differential inclusion

$$-\dot{u}(t) \in \partial I_{C(t)}(u(t)). \tag{2}$$

Here we write ∂I_C for the subdifferential of the characteristic function of a convex set C. In this work, the sets $C(t)$ are assumed to be convex and so $\partial I_{C(t)}$ is a maximal monotone operator depending on time. To solve this problem, J.J. Moreau brings a new important idea in proposing a *catching-up* algorithm. To prove the existence of solutions, he builds discretized solutions in dividing the time interval I into sub-intervals where the convex set C has a little variation. Then by compactness arguments, he shows that a limit mapping can be constructed (when the length of subintervals tends to 0) which satisfies the desired differential inclusion.

Thus it was the first result concerning sweeping process (with no perturbation $F = 0$) because $\partial I_{C(t)}(x) = \mathrm{N}(C(t), x)$ in case of convex sets $C(t)$.

Since then, important improvements have been developed by weakening the assumptions in order to obtain the most general result of existence for sweeping process. There are several directions: one can want to add a perturbation F as written in (1), one may require a weaker assumption than the convexity of the sets $C(t)$, one would like to obtain results in Banach spaces (and not only in Hilbert spaces).