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J. Differential Equations 258 (2015) 1025–1057

Journal of Differential Equations

www.elsevier.com/locate/jde

Stochastic homogenization of interfaces moving with changing sign velocity

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Received 30 May 2014; revised 15 September 2014

Available online 15 November 2014

Abstract

We are interested in the averaging behavior of interfaces moving in stationary ergodic environments with oscillatory normal velocity which changes sign. The problem can be reformulated as the homogenization of a Hamilton–Jacobi equation with a positively homogeneous of degree one non-coercive Hamiltonian. The periodic setting was studied earlier by Cardaliaguet, Lions and Souganidis (2009) [16]. Here we concentrate in the random media and show that the solutions of the oscillatory Hamilton–Jacobi equation converge in L^∞-weak $\star$ to a linear combination of the initial datum and the solutions of several initial value problems with deterministic effective Hamiltonian(s), determined by the properties of the random media.

MSC: 35B27; 70H20; 37A50; 49L25; 78A48

Keywords: Stochastic homogenization; Hamilton–Jacobi equations; Viscosity solutions; Non-coercive Hamiltonian; Random media; Front propagation

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[1] Partially supported by the NSF grants DMS-0901802 and DMS-1266383.
[2] Partially supported by the NSF grant DMS-1361236.

http://dx.doi.org/10.1016/j.jde.2014.09.019
0022-0396/

1. Introduction

We study the averaging behavior of interfaces moving in stationary ergodic environments with oscillatory normal velocity which changes sign. The problem can be reformulated, using the level-set method, as the homogenization of a Hamilton–Jacobi equation with a positively homogeneous of degree one, non-coercive Hamiltonian. The interface $\Gamma^\varepsilon(t,\omega)$ is defined as the zero level set of the solution $u^\varepsilon := u^\varepsilon(x,t,\omega)$ of the initial value problem

$$\begin{cases} u_t^\varepsilon + a\left(\dfrac{x}{\varepsilon},\omega\right)\left|Du^\varepsilon\right| = 0 & \text{in } \mathbb{R}^n \times (0,\infty), \\ u^\varepsilon = u_0 & \text{on } \mathbb{R}^n \times \{0\}, \end{cases} \tag{1.1}$$

where $u_0 \in \mathrm{UC}(\mathbb{R}^n)$, the space of uniformly continuous functions on $\mathbb{R}^n$. The random environment is modeled by a probability space $(\Omega,\mathcal{F},\mathbb{P})$ endowed with an ergodic group of measure preserving transformations $(\tau_z)_{z\in\mathbb{R}^n}$.

The velocity $a := a(y,\omega) : \mathbb{R}^n \times \Omega \to \mathbb{R}$ is a stationary random process with respect to $(\Omega,\mathcal{F},\mathbb{P})$ (precise definitions are given later) and continuous in y for each ω. The main challenge is to analyze the averaging behavior of the u^ε's when $a(\cdot,\omega)$ changes sign. In this case, the Hamiltonian

$$H(p,y,\omega) = a(y,\omega)|p| \tag{1.2}$$

is neither coercive nor convex and, hence, the stochastic homogenization of (1.1) cannot be handled by the theory developed so far for stationary ergodic environments.

When $\inf a = a_0 > 0$, that is when a is strictly positive (a similar analysis applies when $\sup a < 0$), the Hamiltonian is convex and coercive. In this case it is known that there exists a unique positively homogeneous of degree one coercive effective Hamiltonian $\overline{H} = \overline{H}(p)$ so that, as $\varepsilon \to 0$, u^ε converges, locally uniformly in $\mathbb{R}^n \times [0,\infty)$ and almost surely (a.s. for short) in Ω, to the deterministic $\bar{u}$, solution to the effective initial value problem

$$\begin{cases} \bar{u}_t + \overline{H}(D\bar{u}) = 0 & \text{in } \mathbb{R}^n \times (0,\infty), \\ \bar{u} = u_0 & \text{on } \mathbb{R}^n \times \{0\}. \end{cases} \tag{1.3}$$

The main step is, of course, to determine the effective Hamiltonian $\overline{H} = \overline{H}(p)$, which is often called the ergodic constant.

The periodic homogenization of coercive Hamilton–Jacobi and viscous Hamilton–Jacobi equations was first studied by Lions, Papanicolaou and Varadhan [32] and, later, by Evans [20,21] and Majda and Souganidis [35]. Ishii established in [27] the homogenization of Hamilton–Jacobi equations in almost periodic settings. The stochastic homogenization of Hamilton–Jacobi equations with convex and coercive Hamiltonians was established independently by Souganidis [39] and Rezakhanlou and Tarver [37]. In the stochastic setting results for viscous Hamilton–Jacobi equations were obtained by Lions and Souganidis [33] and Kosygina, Rezakhanlou and Varadhan [29], while space–time oscillations were studied by Kosygina and Varadhan [30] and Schwab [38]. In [34] Lions and Souganidis gave a simpler proof for homogenization in probability using weak convergence techniques. Their program was completed by Armstrong and Souganidis in [5,6] using the so-called metric problem. The viscous case was later refined by Armstrong and Tran in [7].