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The diffusive logistic equation with a free boundary and sign-changing coefficient ☆

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Abstract

This short paper concerns a diffusive logistic equation with a free boundary and sign-changing coefficient, which is formulated to study the spread of an invasive species, where the free boundary represents the expanding front. A spreading–vanishing dichotomy is derived, namely the species either successfully spreads to the right-half-space as time $t \to \infty$ and survives (persists) in the new environment, or it fails to establish itself and will extinct in the long run. The sharp criteria for spreading and vanishing are also obtained. When spreading happens, we estimate the asymptotic spreading speed of the free boundary.

MSC: 35K57; 35K61; 35R35; 92D25

Keywords: Diffusive logistic equation; Sign-changing coefficient; Free boundary; Spreading–vanishing; Sharp criteria

1. Introduction

Understanding the nature of establishment and spreading of invasive species is a central problem in invasion ecology. A lot of mathematicians have made efforts to develop various invasion models and investigated them from a viewpoint of mathematical ecology, refer to [2–4,7–18, 22,24–29] for example. Most theoretical approaches are based on or start with single-species models. In consideration of the environmental heterogeneity, the following problem

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$$\begin{cases} u_t - d\Delta u = u\big(m(x) - u\big), & t > 0,\ x \in \Omega, \\ B[u] = 0, & t \ge 0,\ x \in \partial\Omega, \\ u(0, x) = u_0(x), & x \in \Omega \end{cases}$$

is a typical one to describe the spreading of invasive species and has received an astonishing amount of attention, see, for example [2,21] and the references therein. In this model, $u(t, x)$ represents the population density; constant $d > 0$ denotes the diffusion (dispersal) rate; the function $m(x)$ accounts for the local growth rate (intrinsic growth rate) of the population and is positive on favorable habitats and negative on unfavorable ones; Ω is a bounded domain of $\mathbb{R}^N$; the boundary operator $B[u] = \alpha u + \beta \frac{\partial u}{\partial \nu}$, α and β are non-negative functions and $\alpha + \beta > 0$; ν is the outward unit normal vector of the boundary $\partial\Omega$. The corresponding systems with heterogeneous environment have also been studied extensively, please refer to [3,4,18,21] and the references cited therein.

To realize the spreading mechanism of an invading species (how fast the species spreads into new territory, and what factors influence the successful spread), Du and Lin [11] proposed the following free boundary problem of the diffusive logistic equation

$$\begin{cases} u_t - du_{xx} = u(a - bu), & t > 0,\ 0 < x < h(t), \\ u_x(t, 0) = 0, \quad u\big(t, h(t)\big) = 0, & t \ge 0, \\ h'(t) = -\mu u_x\big(t, h(t)\big), & t \ge 0, \\ h(0) = h_0, \quad u(0, x) = u_0(x), & 0 \le x \le h_0, \end{cases} \tag{1.1}$$

where $x = h(t)$ is the moving boundary to be determined; a, b, d, h_0 and μ are given positive constants, h_0 denotes the size of initial habitat, μ is the ratio of expanding speed of the free boundary and population gradient at expanding front, it can also be considered as the "moving parameter"; u_0 is a given positive initial function. They have derived various interesting results.

Since then, this kind of problem describing the spread by free boundary has been studied intensively. For example, when the boundary condition $u_x = 0$ at $x = 0$ in (1.1) is replaced by $u = 0$, such free boundary problem was studied by Kaneko and Yamada [17]. Du and Guo [7,8], Du, Guo and Peng [9] and Du and Liang [10] considered the higher space dimensions, heterogeneous environment and time-periodic environment case, where the heterogeneous environment coefficients were required to have positive lower and upper bounds. Peng and Zhao [22] studied the seasonal succession case. When the nonlinear term $u(a - bu)$ is replaced by a general function $f(u)$, this problem has been investigated by Du and Lou [13] and Du, Matsuzawa and Zhou [14]. The diffusive competition system with a free boundary has been studied by Guo and Wu [15], Du and Lin [12] and Wang and Zhao [26]. The diffusive prey–predator model with free boundaries has been studied by Wang and Zhao [24,25,28].

Recently, Zhou and Xiao [29] studied the following diffusive logistic model with a free boundary in heterogeneous environment:

$$\begin{cases} u_t - du_{xx} = u\big(m(x) - u\big), & t > 0,\ 0 < x < h(t), \\ u_x(t, 0) = 0, \quad u\big(t, h(t)\big) = 0, & t \ge 0, \\ h'(t) = -\mu u_x\big(t, h(t)\big), & t \ge 0, \\ h(0) = h_0, \quad u(0, x) = u_0(x), & 0 \le x \le h_0, \end{cases}$$