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# Asymptotic behavior on a kind of parabolic Monge–Ampère equation

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**Abstract**

In this paper, we apply level set and nonlinear perturbation methods to obtain the asymptotic behavior of the solution to a kind of parabolic Monge–Ampère equation at infinity. The Jörgens–Calabi–Pogorelov theorem for parabolic and elliptic Monge–Ampère equation can be regarded as special cases of our result.


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*Keywords:* Parabolic Monge–Ampère equation; Asymptotic behavior; Level set method; Nonlinear perturbation method

## 1. Introduction and main results

In affine geometry, a well known theorem of Jörgens ($n = 2$ [11]), Calabi ($n \leq 5$ [4]) and Pogorelov ($n \geq 2$ [14]) asserts that a convex improper affine hypersurface is an elliptic paraboloid. This theorem can also be stated as follows: any classical convex solution of the elliptic Monge–Ampère equation

$$\det(D^2 u) = 1 \quad \text{in } \mathbb{R}^n$$

must be a quadratic polynomial.

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Along the lines of affine geometry, a simpler and more analytic proof was given by Cheng and Yau in [6]. Jost and Xin also give another proof of this result in [12]. Caffarelli proved that Jörgens–Calabi–Pogorelov theorem remains valid for viscosity solution in [2].

In [8], Gutiérrez and Huang established Jörgens–Calabi–Pogorelov theorem to the following kind of parabolic Monge–Ampère equation

$$-u_t \det(D^2u) = 1 \quad \text{in } \mathbb{R}^{n+1}_- := \mathbb{R}^n \times (-\infty, 0], \quad -M_1 \le u_t \le -M_2,$$

where $M_1$ and $M_2$ are two positive constants. In [17], Xiong and Bao extended Jörgens–Calabi–Pogorelov theorem to more general parabolic Monge–Ampère equations of the form

$$u_t = \rho\left(\log(\det(D^2u))\right) \quad \text{in } \mathbb{R}^{n+1}_-,$$

which covers the results in [8].

In [3], Caffarelli and Li obtained the asymptotic behavior of convex viscosity solutions of

$$\det(D^2u) = f(x) \quad \text{in } \mathbb{R}^n,$$

where $f \in C^0(\mathbb{R}^n)$ satisfies

$$0 \le \inf_{\mathbb{R}^n} f \le \sup_{\mathbb{R}^n} f < \infty,$$
$$\text{support } (f-1) \text{ is bounded.}$$

Recently, Zhang, Wang and Bao [18] have extended the above result to the following parabolic Monge–Ampère equation

$$-u_t \det(D^2u) = f(x,t) \quad \text{in } \mathbb{R}^{n+1}_-.$$

In this paper, we investigate classical solutions to the parabolic Monge–Ampère equation

$$u_t - \log\left(\det(D^2u)\right) = f(x,t) \quad \text{in } \mathbb{R}^{n+1}_-, \tag{1}$$

such that there exist two constants $c_0$ and $C_0$ with

$$c_0 \le u_t \le C_0 \quad \text{in } \mathbb{R}^{n+1}_-, \tag{2}$$

where $f \in C^0(\mathbb{R}^{n+1}_-)$ and there exists $a \in \mathbb{R}^1$ such that

$$\text{support } (f-a) \text{ is bounded.} \tag{3}$$

In the following theorem, we obtain the asymptotic behavior of solutions of (1) and (2) under the hypothesis (3).