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ScienceDirect
J. Differential Equations 259 (2015) 2546–2575
Journal of Differential Equations
www.elsevier.com/locate/jde

Continuous subsonic–sonic flows in a general nozzle ☆

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Received 17 August 2014
Available online 29 April 2015

Abstract

This paper concerns continuous subsonic–sonic potential flows in a two dimensional finite nozzle with a general upper wall and a straight lower wall. We give a class of nozzles where continuous subsonic–sonic flows may exist. Consider a continuous subsonic–sonic flow in such a nozzle after rescaling the upper wall in a small scale. It is shown that for a given inlet and a fixed point at the upper wall, there exists uniquely a continuous subsonic–sonic flow whose velocity vector is along the normal direction at the inlet and the sonic curve, which satisfies the slip conditions on the nozzle walls and whose sonic curve intersects the upper wall at the fixed point. Furthermore, the sonic curve of this flow is a free boundary, where the flow is singular in the sense that the speed is only $C^{1/2}$ Hölder continuous and the acceleration blows up at the sonic state. As the scale tends to zero, the precise convergent rate of the continuous subsonic–sonic flow converging to the sonic state is also determined.

MSC: 35R35; 35Q35; 76N10; 35J70

Keywords: Continuous subsonic–sonic flow; Free boundary; Degeneracy; Singularity

1. Introduction

In this paper, we study the isentropic, irrotational, steady compressible Euler subsonic–sonic flows in general nozzles, which arise in the physical experiments and the engineering designs.

☆ Supported by the National Natural Science Foundation of China (Grant No. 1107222106).
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http://dx.doi.org/10.1016/j.jde.2015.03.036

Roughly speaking, there have been mainly two kinds of problems on subsonic–sonic flows. One is subsonic–sonic flows past a profile. In the significant work [1], Bers showed that for two dimensional potential flows past a profile, the whole flow field will be subsonic outside the profile if the Mach number of the freestream is small enough; furthermore, the maximum flow speed will tend to the sound speed as the freestream Mach number increases. Bers [1] did not consider the flow with the critical freestream Mach number. Based on the compensated compactness method, it was shown in [3] that the flows with sonic points past a profile may be realized as weak limits of sequences of strictly subsonic flows. The other kind is subsonic–sonic flows in an infinite nozzle. Bers [2] asserted that there is a global subsonic potential flow through an infinite nozzle for an appropriately given incoming mass flux, which was proved rigorously in [10]. It was shown that there exists a critical value for a general infinite nozzle such that a strictly subsonic flow exists uniquely as long as the incoming mass flux is less than the critical value. Furthermore, a class of subsonic–sonic flows can be obtained as the weak limits of strictly subsonic flows associated with the incoming mass fluxes increasing to the critical value. Moreover, Xie and Xin [11] considered subsonic–sonic flows in axially symmetric nozzles. It should be noted that the subsonic–sonic flows past a profile and the ones in an infinite nozzle obtained in [3,10] are both in the weak sense and have not been proved to be smooth yet, thus the locations and properties of the sonic states are unknown for such flows. A typical subsonic–sonic flow with precise regularity is a symmetric continuous subsonic–sonic flow in a finite straight nozzle, whose structural stability was studied in the recent works [7,8]. Precisely, for the given inlet, which is a perturbation of an arc centered at the vertex of the nozzle, and the given incoming mass flux, it was shown in [8] that there exists an open interval depending only on the adiabatic exponent and the length of the arc, such that a unique continuous subsonic–sonic flow, whose velocity vector is along the normal direction at the inlet and the outlet and which satisfies the slip conditions on the nozzle walls, exists as long as the incoming mass flux belongs to this interval and the perturbation of the inlet is sufficiently small; furthermore, the sonic curve of this continuous subsonic–sonic flow is a free boundary, where the flow is singular in the sense that the speed is only $C^{1/2}$ Hölder continuous and the acceleration blows up at the sonic state. Furthermore, Wang and Xin [9] studied smooth transonic flows of Meyer type in finite de Laval nozzles and showed that there exists uniquely a smooth transonic flow near the throat of the nozzle, whose sonic curve is located at the throat and whose acceleration is Lipschitz continuous, if the wall of the nozzle is sufficiently flat.

It is shown from [8] that the existence of continuous subsonic–sonic flows depends on the geometry of the nozzle. A natural question is how to formulate the continuous subsonic–sonic flow problem in a general nozzle. In [8], the authors prescribed the incoming flow angle and the incoming mass flux, which are two physical quantities, to formulate the continuous subsonic–sonic flow problem in a straight nozzle; furthermore, the sonic curve of the continuous subsonic–sonic flow is a free boundary where the velocity vector is along the normal direction. Clearly, for a continuous subsonic–sonic flow whose velocity vector is along the normal direction at the sonic curve, the second derivative of the nozzle wall at the intersecting point with the sonic curve must be zero. Therefore, it is not suitable to prescribe the incoming flow angle and the incoming mass flux to formulate the continuous subsonic–sonic flow problem in a general nozzle. In the present paper, we fix the intersecting point between the sonic curve and the upper wall to replace the incoming mass flux condition. That is to say, for a general nozzle with a given inlet, we seek a continuous subsonic–sonic flow whose velocity vector is along the normal direction at the inlet and the sonic curve, which satisfies the slip conditions on the nozzle walls and whose sonic curve intersects the upper wall at a fixed point. Furthermore, the sonic curve of the flow, which is chosen to be the outlet of the flow, is a free boundary.