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On some transmission problems set in a biological cell, analysis and resolution ☆

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Abstract

Some transmission problems are set in bodies with a crown of small thickness $\varepsilon > 0$. For instance, those concerning the conductivity in the biological cell. By a natural change of variables, we transform them in transmission problems set in two cylindrical bodies $]-\infty, 0[\times]-\pi, \pi[$ and $]0, \delta[\times]-\pi, \pi[$ (where $\delta = \ln(1+\varepsilon)$) and then, in some general elliptic abstract differential equations $\left(P^{\delta}\right)_{\delta>0}$. The goal of this work is to give a complete study of these problems $\left(P^{\delta}\right)_{\delta>0}$ for every $\delta > 0$. Existence, uniqueness and maximal regularity results are obtained for the classical solutions essentially by using the semigroups theory.

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1. Introduction

Let us consider the model of a biological cell, constituted of a homogeneous cytoplasm Ω_-^* (of boundary Γ^*) centered at $(0,0)$ with radius of one micrometer surrounded by a thin membrane $\Omega_+^{*\varepsilon}$ (of boundary $\Gamma_+^{*\varepsilon}$) with thickness of few nanometers $\varepsilon > 0$. The electric potential in this cell $\Omega^{*\varepsilon} = \overline{\Omega_-^*} \cup \Omega_+^{*\varepsilon}$ verifies the following problem

$$\left(P^\varepsilon\right) \begin{cases} \nabla.\left(\mu \nabla w^\varepsilon\right) = \mu h^\varepsilon & \text{in } \Omega^{*\varepsilon} \\ \dfrac{\partial w^\varepsilon}{\partial n} = l_+^\varepsilon & \text{on } \Gamma_+^{*\varepsilon} \\ \displaystyle\int_{\Gamma^*} w^\varepsilon(\sigma)\, d\sigma = 0, \end{cases}$$

where

$$\mu = \begin{cases} \mu_- & \text{in } \Omega_-^* \\ \mu_+ & \text{in } \Omega_+^{*\varepsilon} \end{cases}$$

(typically about 1 S/m (Siemens per meter), 5×10^{-7} S/m respectively) are the conductivity positive coefficients of the two bodies Ω_-^*, $\Omega_+^{*\varepsilon}$ depending possibly on ε, and the electric charge density

$$h^\varepsilon = \begin{cases} h_- & \text{in } \Omega_-^* \\ h_+^\varepsilon & \text{in } \Omega_+^{*\varepsilon} \end{cases}$$

is taken, for instance, in space $L^p(\Omega^{*\varepsilon})$, $1 < p < \infty$, that is $h_- \in L^p\left(\Omega_-^*\right)$, $h_+^\varepsilon \in L^p\left(\Omega_+^{*\varepsilon}\right)$ and $\partial/\partial n$ denotes the outward normal derivative, l_+^ε is the electric field imposed on the boundary $\Gamma_+^{*\varepsilon}$. The Neumann boundary condition on $\Gamma_+^{*\varepsilon}$ implies the following compatibility condition on l_+^ε and (μh^ε)

$$\int_{\Omega^{*\varepsilon}} \left(\mu h^\varepsilon\right)(x,y)\, dx dy + \int_{\Gamma_+^{*\varepsilon}} \mu_+ l_+^\varepsilon(\sigma)\, d\sigma = 0.$$

The gauge condition

$$\int_{\Gamma^*} w^\varepsilon(\sigma)\, d\sigma = 0$$

is imposed to have the uniqueness of the solution. Problem (P^ε) can be written in the form