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On axially symmetric incompressible magnetohydrodynamics in three dimensions

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Abstract

In the short article we study the ideal incompressible magnetohydrodynamic equations in three dimensions in which the Faraday law is inviscid. We prove the global well-posedness of classical solutions for a family of special axisymmetric initial data whose swirl components of the velocity field and magnetic vorticity field are trivial.

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1. Introduction

This short paper is aimed to construct a family of global classical solutions to the ideal magnetohydrodynamic system in 3D. Denote a point in $\mathbb{R}^3$ by x. Let $r = \sqrt{x_1^2 + x_2^2}$ and

$$e_r = (x_1/r, x_2/r, 0)^\top, \qquad e_\theta = (-x_2/r, x_1/r, 0)^\top, \qquad e_z = (0, 0, 1)^\top.$$

It is well-known that the 3D incompressible Navier–Stokes equation admits local classical axi-symmetric solutions (see for instance, [17])

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$$v(t,x) = v^r(t,r,z)e_r + v^\theta(t,r,z)e_\theta + v^z(t,r,z)e_z, \qquad p(t,x) = p(t,r,z).$$

However, the global well-posedness of this family of axisymmetric solutions is still widely open.

Denote

$$u(t,x) = v^r(t,r,z)e_r + v^z(t,r,z)e_z, \qquad h(t,x) = v^\theta(t,r,z)e_\theta. \tag{1.1}$$

In terms of (u,h) in (1.1), it is not difficult to write the axisymmetric Navier–Stokes equation in the following form:

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla p = \Delta u - h\cdot\nabla h, \\ \partial_t h + u\cdot\nabla h = \Delta h - h\cdot\nabla u, \\ \nabla\cdot u = 0, \quad \nabla\cdot h = 0. \end{cases} \tag{1.2}$$

By changing two signs in the Navier–Stokes equation (1.2), we can get the following magnetohydrodynamic system (here we even ignored the magnetic diffusion, which is what we mean "ideal" throughout this paper)

$$\begin{cases} \partial_t u + u\cdot\nabla u + \nabla p = \Delta u + B\cdot\nabla B, \\ \partial_t B + u\cdot\nabla B = B\cdot\nabla u, \\ \nabla\cdot u = 0, \quad \nabla\cdot B = 0. \end{cases} \tag{1.3}$$

It is not surprising for experts that the ideal magnetohydrodynamic system (1.3) also admits axisymmetric local classical solutions of the special form

$$u(t,x) = u^r(t,r,z)e_r + u^z(t,r,z)e_z, \qquad B(t,x) = B^\theta(t,r,z)e_\theta. \tag{1.4}$$

In this article, we will prove that this family solutions are global. To the best of our knowledge, this provides the first example of global large solution with finite energy for ideal MHD system (1.3).

More precisely, we prove the following theorem:

Theorem 1.1. *Suppose that u_0 and B_0 are both axially symmetric divergence-free vectors with $u_0^\theta = 0$ and $B_0^r = B_0^z = 0$. Moreover, we assume that $(u_0, B_0) \in H^s$ with $s \geq 2$ and $\frac{B_0^\theta}{r} \in L^\infty$. Then there exists a unique global solution (u,B) for the ideal MHD system (1.3) with the initial data (u_0, B_0) which satisfies*

$$\|u(t,\cdot)\|_{H^2}^2 + \|B(t,\cdot)\|_{H^2}^2 + \int_0^t \|\nabla u\|_{H^2}^2 ds \lesssim C_0 e^{C_0 e^{C_0(1+t)^{\frac{7}{4}}} e^{C_0 t^{\frac{5}{4}}}},$$

where C_0 is a positive constant depending only on the H^2 norm of u_0 and B_0.

Remark 1.2. Clearly, it is not hard to extend the above result to the fully resistive MHD using similar observations.

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