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J. Differential Equations 259 (2015) 3344–3365

Journal of Differential Equations

www.elsevier.com/locate/jde

Logarithmic stability in determining two coefficients in a dissipative wave equation. Extensions to clamped Euler–Bernoulli beam and heat equations

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Received 8 November 2014; revised 31 March 2015

Available online 27 April 2015

Abstract

We are concerned with the inverse problem of determining both the potential and the damping coefficient in a dissipative wave equation from boundary measurements. We establish stability estimates of logarithmic type when the measurements are given by the operator who maps the initial condition to Neumann boundary trace of the solution of the corresponding initial–boundary value problem. We build a method combining an observability inequality together with a spectral decomposition. We also apply this method to a clamped Euler–Bernoulli beam equation. Finally, we indicate how the present approach can be adapted to a heat equation.

MSC: 93C25; 93B07; 93C20; 35R30

Keywords: Damping coefficient; Potential; Dissipative wave equation; Boundary measurements; Boundary observability; Initial-to-boundary operator

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http://dx.doi.org/10.1016/j.jde.2015.04.023

0022-0396/

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1. Introduction

We consider the following initial–boundary value problem (abbreviated to IBVP in the sequel) for the wave equation:

$$\begin{cases} \partial_t^2 u - \Delta u + q(x)u + a(x)\partial_t u = 0 & \text{in } Q = \Omega \times (0,\tau), \\ u = 0 & \text{on } \Sigma = \partial\Omega \times (0,\tau), \\ u(\cdot,0) = u_0,\ \partial_t u(\cdot,0) = u_1, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$, $n \ge 1$, is a bounded domain with C^2-smooth boundary $\partial\Omega$ and $\tau > 0$.

We assume in this text that the coefficients q and a are real-valued.

Under the assumption that $q, a \in L^\infty(\Omega)$, for each $\tau > 0$ and $\begin{pmatrix} u_0 \\ u_1 \end{pmatrix} \in H_0^1(\Omega) \times L^2(\Omega)$, the IBVP (1.1) has a unique solution $u_{q,a} \in C([0,\tau], H_0^1(\Omega))$ such that $\partial_t u_{q,a} \in C([0,\tau], L^2(\Omega))$ (e.g. [6, pages 699–702]). On the other hand, by a classical energy estimate, we have

$$\|u_{q,a}\|_{C([0,\tau],H_0^1(\Omega))} + \|\partial_t u_{q,a}\|_{C([0,\tau],L^2(\Omega))} \le C(\|u_0\|_{1,2} + \|u_1\|_0).$$

Here and henceforth, $\|\cdot\|_p$ and $\|\cdot\|_{s,p}$, $1 \le p \le \infty$, $s \in \mathbb{R}$, denote respectively the usual L^p-norm and the $W^{s,p}$-norm.

We note that the constant C above is a nondecreasing function of $\|q\|_\infty + \|a\|_\infty$.

Now, since $u_{q,a}$ coincides with the solution of the IBVP (1.1) in which $-q(x)u_{q,a} - a(x)\partial_t u_{q,a}$ is seen as a right-hand side, we can apply [14, Theorem 2.1] to get that $\partial_\nu u_{q,a}$, the derivative of the $u_{q,a}$ in the direction of ν, the unit outward normal vector to $\partial\Omega$, belongs to $L^2(\Sigma)$. Additionally, the mapping

$$\begin{pmatrix} u_0 \\ u_1 \end{pmatrix} \in H_0^1(\Omega) \times L^2(\Omega) \longrightarrow \partial_\nu u_{q,a} \in L^2(\Sigma)$$

defines a bounded operator.

Let Γ be a nonempty open subset of $\partial\Omega$ and $\Upsilon = \Gamma \times (0,\tau)$. To $q, a \in L^\infty(\Omega)$, we associate the initial-to-boundary (abbreviated to IB in the following) operator $\Lambda_{q,a}$ defined by