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Regular traveling waves for a nonlocal diffusion equation

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Abstract

In this paper, we study a nonlocal diffusion equation with a general diffusion kernel and delayed nonlinearity, and obtain the existence, nonexistence and uniqueness of the regular traveling wave solutions for this nonlocal diffusion equation. As an application of the results, we reconsider some models arising from population dynamics, epidemiology and neural network. It is shown that there exist regular traveling wave solutions for these models, respectively. This generalized and improved some results in literatures.
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1. Introduction

In this paper, we consider the following nonlocal diffusion equation

$$u_t(t,x) = (J_\rho * u - u)(t,x) + f\big(u, G ** S(u)\big)(t,x), \quad t \in \mathbb{R}, x \in \mathbb{R}, \tag{1.1}$$

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where $J_\rho(\cdot)$ is a ρ-parameterized symmetric kernel given by $\frac{1}{\rho}J(\frac{y}{\rho})$, parameter ρ represents the nonlocal dispersal distance if $\rho > 0$ and no dispersal if $\rho = 0$ (here $J_0(\cdot) \triangleq \lim_{\rho\to 0^+} \frac{1}{\rho}J(\frac{y}{\rho}) = \delta(y)$, see[20]), $J \in L^1(\mathbb{R})$ is a positive even function with $\int_{\mathbb{R}} J(y)dy = 1$. And $\int_{\mathbb{R}} J(y)e^{-\sigma y}dy$ is convergent for $\sigma \in [0, \chi)$, while it approaches to infinity as $\sigma \to \chi^-$, where χ could be finite or infinite. Two typical examples are

$$J(y) = \frac{1}{2}e^{-|y|} \quad \text{and} \quad J(y) = \frac{1}{\sqrt{\pi}}e^{-y^2}.$$

The convolution terms are defined by

$$(J_\rho * u - u)(t, x) = \int_{\mathbb{R}} J_\rho(x - y)\big[u(t, y) - u(t, x)\big]dy$$

and

$$G ** S(u)(t, x) = \int_{-\infty}^{t} \int_{\mathbb{R}} G(t - s, x - y)S\big(u(s, y)\big)dyds = \int_{0}^{\infty} \int_{\mathbb{R}} G(s, y)S\big(u(t - s, x - y)\big)dyds.$$

Eq. (1.1) is called *nonlocal diffusion equation* since the diffusion process is modelled by a nonlocal operator $\mathcal{N}u := (J_\rho * u - u)(x, t)$, which describes that the diffusion of density u at a point x and time t depends not only on $u(x, t)$ but also on all the values of u in a neighborhood of x through the convolution term $J_\rho * u$. In population dynamics, the reaction term $f(u, G ** S(u))$ is usually used to describe the recruits of population, S is the birth rate of species during the dispersal process, and $G ** S(u)$ represents a weighted average of the population density both in past time and space [2].

Since Eq. (1.1) involves a general diffusion kernel and delayed nonlinearity, it can be reduced to some well-known equations if ρ and $G(t, x)$ are taken as some special forms.

(i) If $\rho = 0$, we have $J_\rho(x) = \delta(x)$. Then (1.1) reduces to the integro-differential equation

$$u_t(t, x) = f\big(u, G ** S(u)\big)(t, x). \tag{1.2}$$

Under the assumption that S is a diffeomorphism on a given bounded interval of $\mathbb{R}$ such that $S' > 0$, Schumacher [19] showed that (1.2) has a unique regular traveling wave solution, here a traveling wave solution $u(t, x) = U(x + ct)$ is called the regular traveling wave solution if there exists a positive constant β such that $\lim_{\xi\to-\infty} U(\xi)e^{-\beta\xi}$ exists and is positive (see Definition 2 on page 61 in [19]).

(ii) If $\rho = 0$, $f(u, v) = -u + v$ and $G(t, x) = \delta(t)k(x)$, then (1.1) can be reduced to the equation

$$u_t(t, x) = -u(t, x) + \int_{\mathbb{R}} k(x - y)S\big(u(t, y)\big)dy. \tag{1.3}$$

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