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Concentration through large advection [☆]

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Abstract

In this paper we extend the elegant results of Chen, Lam and Lou [6, Section 2], where a concentration phenomenon was established as the advection blows up, to a general class of adventive–diffusive generalized logistic equations of degenerate type. Our improvements are really sharp as we allow the carrying capacity of the species to vanish in some subdomain with non-empty interior. The main technical devices used in the derivation of the concentration phenomenon are Proposition 3.2 of Cano-Casanova and López-Gómez [5], Theorem 2.4 of Amann and López-Gómez [1] and the classical Harnack inequality. By the relevance of these results in spatial ecology, complete technical details seem imperative, because the proof of Theorem 2.2 of [6] contains some gaps originated by an "optimistic" use of Proposition 3.2 of [5]. Some of the general assumptions of [6] are substantially relaxed.

MSC: primary 35B25, 35B36, 35B40; secondary 92D25

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1. Introduction

This paper studies the effect of a large advection α on the dynamics of the parabolic problem

$$\begin{cases} \partial_t u = \nabla \cdot (D\nabla u - \alpha u \nabla m) + u\big(\lambda m - a u^{p-1}\big) & \text{in } \Omega,\ t > 0, \\ D\partial_n u - \alpha u \partial_n m = 0 & \text{on } \partial\Omega,\ t > 0, \\ u(\cdot, 0) = u_0 > 0 & \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is a smooth bounded domain (open and connected set) of $\mathbb{R}^N$, $N \geq 1$, $D > 0$, $\alpha > 0$, $\lambda \in \mathbb{R}$, $p \geq 2$, $a \in \mathcal{C}(\bar{\Omega})$ satisfies $a > 0$, in the sense that $a \geq 0$ and $a \not\equiv 0$, n stands for the outward unit normal along the boundary of Ω, denoted by $\partial\Omega$, and $m \in \mathcal{C}^2(\bar{\Omega})$ is a function such that

$$m(x_+) > 0, \qquad m(x_-) < 0 \quad \text{for some } x_+, x_- \in \Omega. \tag{1.2}$$

The initial data u_0 are assumed to be in $L^\infty(\Omega)$. Under these conditions, it is well known that there exists $T > 0$ such that (1.1) admits a unique classical solution, denoted by $u(x, t; u_0)$ in $[0, T]$ (see, e.g., Henry [9], Daners and Koch [8] and Lunardi [14]). Moreover, the solution is unique if it exists, and according to the parabolic strong maximum principle of Nirenberg [15], $u(\cdot, t; u_0) \gg 0$ in Ω, in the sense that

$$u(x, t; u_0) > 0 \quad \text{for all } x \in \bar{\Omega} \text{ and } t \in (0, T].$$

Thus, since $a > 0$ in Ω, we have that

$$\partial_t u = \nabla \cdot (D\nabla u - \alpha u \nabla m) + u\big(\lambda m - a u^{p-1}\big) \leq \nabla \cdot (D\nabla u - \alpha u \nabla m) + \lambda m u$$

and, hence, thanks again to the parabolic maximum principle,

$$u(\cdot, t; u_0) \ll v(\cdot, t; u_0) \quad \text{for all } t \in (0, T],$$

where $v(x, t; u_0)$ stands for the unique solution of the linear parabolic problem

$$\begin{cases} \partial_t v = \nabla \cdot (D\nabla v - \alpha v \nabla m) + \lambda m v & \text{in } \Omega,\ t > 0, \\ D\partial_n v - \alpha v \partial_n m = 0 & \text{on } \partial\Omega,\ t > 0, \\ v(\cdot, 0) = u_0 > 0 & \text{in } \Omega. \end{cases}$$

As v is globally defined in time, $u(x, t; u_0)$ cannot blow up in a finite time and, therefore, it is globally defined for all $t > 0$. In applications it is imperative to characterize the asymptotic behavior of $u(x, t; u_0)$ as $t \uparrow \infty$.

A special version of this model (with $\lambda = 1$, $p = 2$ and $a(x) > 0$ for all $x \in \bar{\Omega}$) was introduced by Belgacem and Cosner [4] to "*analyze the effects of adding a term describing drift or advection along environmental gradients to reaction diffusion models for population dynamics with dispersal*". In these models, $u(x, t)$ stands for the density of a population at time $x \in \Omega$ after time $t > 0$, $D > 0$ is the usual diffusion rate, and "*the constant α measures the rate at which the population moves up the gradient of the growth rate $m(x)$. If $\alpha < 0$, the population would move*