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Asymptotic stability of the compressible gas–liquid model with well-formation interaction and gravity

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Abstract

In this paper we are concerned with the initial–boundary value problem of the compressible gas–liquid model with well-formation interaction and gravity. The asymptotic behavior of solutions to steady states is established. Also the time-decay rates of perturbed solutions in the sense of L^∞ norm are obtained under some suitable assumptions on the initial date, if $\gamma > 1$ (associated with pressure law of gas) and $\beta \in (0, \frac{\gamma}{2}] \cap (0, \gamma - \alpha\gamma) \cap (0, \frac{\gamma+\alpha\gamma}{3}]$ where β characterizes the viscosity coefficient and α describes the mass decay rate at the boundary. A main purpose of this work is to clarify the role played by the well-reservoir interaction term. The analysis demonstrates that it is essential to take into account information about sign as well as size of the interaction term in order to obtain time-independent estimates when it operates in combination with gravity.

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Contents

1. Introduction

The compressible drift-flux gas–liquid model is often used in chemical engineering to describe the dynamics of two-phase flow, see [1,2]. This model is different from the two-fluid model in the sense that it has one mixture momentum equation instead of two separate momentum equations. We refer to [10] for more on the relationship between the two-fluid and drift-flux models. In this paper, we are concerned with a gas–liquid model where gas is allowed to flow between a wellbore and surrounding formation governed by a given function $A(x,t)$. From an application point of view, $A(x,t) > 0$ means that there is inflow of gas along the well and $A(x,t) < 0$ means that there is outflow of gas along the well, see [4]. More precisely, the corresponding model can be written in Eulerian coordinates as

$$\begin{cases} \partial_t n + \partial_x [nu] = nA(x,t), \\ \partial_t m + \partial_x [mu] = 0, \\ \partial_t \big[(m+n)u\big] + \partial_x \big[(m+n)u^2\big] + \partial_x P = gm + \partial_x [\varepsilon \partial_x u], \quad a(t) < x < b, \end{cases} \tag{1.1}$$

where the free boundary function $a(t)$ satisfies

$$\begin{cases} \dfrac{da(t)}{dt} = u\big(a(t),t\big), \quad t > 0, \\ a(0) = a. \end{cases}$$

Here, $n = n(x,t) \geq 0$, $m = m(x,t) \geq 0$ respectively are masses of the gas and liquid. $u = u(x,t)$ denotes velocity of the phases. Initial data is given as

$$m(x,0) = m_0(x), \qquad n(x,0) = n_0(x), \qquad u(x,0) = u_0(x), \tag{1.2}$$

and the boundary conditions

$$n\big(a(t),t\big) = 0, \qquad m\big(a(t),t\big) = 0, \qquad u(b,t) = 0, \quad t > 0. \tag{1.3}$$

The pressure function $P(\cdot,\cdot)$ and viscosity function $\varepsilon(\cdot,\cdot)$ depending on the masses satisfy

$$P(n,m) = K_1 \left(\frac{n}{\rho_l - m} \right)^{\gamma}, \qquad \varepsilon(n,m) = \frac{K_2 n^{\beta}}{(\rho_l - m)^{\beta+1}}, \quad K_1, K_2 > 0. \tag{1.4}$$

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