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On the equations of thermally radiative magnetohydrodynamics

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Abstract

An initial–boundary value problem is considered for the viscous compressible thermally radiative magnetohydrodynamic (MHD) flows coupled to self-gravitation describing the dynamics of gaseous stars in a bounded domain of $\mathbb{R}^3$. The conservative boundary conditions are prescribed. Compared to Ducomet–Feireisl [13] (also see, for instance, Feireisl [18], Feireisl–Novotný [20]), a rather more general constitutive relationship is given in this paper. The analysis allows for the initial density with vacuum. Every transport coefficient admits a certain temperature scaling. The global existence of a variational (weak) solution with any finite energy and finite entropy data is established through a three-level approximation and methods of weak convergence.

Keywords: Magnetohydrodynamic (MHD) flows; Compressible; Thermally radiative; Global existence; Variational (weak) solution

1. Introduction

Magnetohydrodynamics (MHD) concerns the motion of conducting fluids (cf. gases) in an electromagnetic field with a very broad range of applications in physical areas from liquid metals to cosmic plasmas. In moving conducting magnetic fluids, magnetic fields can induce electric fields, and electric currents are developed, which create forces on the fluids and considerably

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affect changes in the magnetic fields. The dynamic motion of the fluids and the magnetic field interact strongly with each other and both the hydrodynamic and electrodynamic effects have to be taken into account. Except for this, considerable attention has been put to study the effects of thermal radiation recently, because the radiation field significantly affects the dynamics of fluids, for example, certain re-entry of space vehicles, astrophysical phenomena and nuclear fusion, and hydrodynamics with explicit account of radiation energy and momentum contribution constitutes the character of radiation hydrodynamics. In this paper, we consider the viscous compressible thermally radiative conducting fluids driven by the self-gravitation in the full magnetohydrodynamic setting. The equations to the three-dimensional full magnetohydrodynamic flows have the following form [3,13,29,30]:

$$\begin{cases} \rho_t + \nabla\cdot(\rho\mathbf{u}) = 0, \quad \mathbf{x}\in\Omega\subset R^3,\ t>0, \\ (\rho\mathbf{u})_t + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla p = \nabla\cdot\mathbb{S} + \rho\nabla\Psi + (\nabla\times\mathbf{H})\times\mathbf{H}, \\ \mathcal{E}_t + \nabla\cdot\left(\left(\rho e + \dfrac{1}{2}\rho|\mathbf{u}|^2 + p\right)\mathbf{u}\right) + \nabla\cdot\mathbf{q} \\ \quad = \nabla\cdot\big((\mathbf{u}\times\mathbf{H})\times\mathbf{H} + \nu\mathbf{H}\times(\nabla\times\mathbf{H}) + \mathbb{S}\mathbf{u}\big) + \rho\nabla\Psi\cdot\mathbf{u}, \\ \mathbf{H}_t - \nabla\times(\mathbf{u}\times\mathbf{H}) = -\nabla\times(\nu\nabla\times\mathbf{H}), \quad \nabla\cdot\mathbf{H} = 0, \end{cases} \tag{1.1}$$

where $\rho\in\mathbb{R}$ denotes the density, $\mathbf{u}\in\mathbb{R}^3$ the fluid velocity and $\mathbf{H}\in\mathbb{R}^3$ the magnetic field, $p\in\mathbb{R}$ the pressure.

$$\mathcal{E} = \rho e + \frac{1}{2}\big(\rho|\mathbf{u}|^2 + |\mathbf{H}|^2\big)$$

is the total energy with e being the specific internal energy. $\mathbb{S}$ stands for the viscous stress tensor, given by Newton's law of viscosity:

$$\mathbb{S} = \mu\big(\nabla\mathbf{u} + \nabla^\top\mathbf{u}\big) + \lambda(\nabla\cdot\mathbf{u})\mathbb{I}_3 \tag{1.2}$$

with μ the shear viscosity coefficient and $\eta = \lambda + \frac{2}{3}\mu$ the bulk viscosity coefficient of the flow (while μ should be positive for any "genuinely" viscous fluid, η may vanish, e.g. for a monoatomic gas), $\mathbb{I}_3$ the 3×3 identity matrix and $\nabla^\top\mathbf{u}$ the transpose of the matrix $\nabla\mathbf{u}$. Note that

$$\nabla\cdot\mathbb{S} = \left(\eta + \frac{1}{3}\mu\right)\nabla(\nabla\cdot\mathbf{u}) + \mu\Delta\mathbf{u},$$

$$\mathbb{S}:\nabla\mathbf{u} = \mu|\nabla\mathbf{u}|^2 + \mu\nabla\mathbf{u}:\nabla^\top\mathbf{u} + \left(\eta - \frac{2}{3}\mu\right)(\nabla\cdot\mathbf{u})^2.$$

$\mathbf{q}$ is the heat flux obeying the classical Fourier's law:

$$\mathbf{q} = -\kappa\nabla\vartheta, \quad \kappa\geq 0, \tag{1.3}$$

where ϑ means the absolute temperature, κ is the heat conductivity coefficient. The term $\rho\nabla\Psi$ is the gravitational force where the potential Ψ obeys Poisson's equation on the whole physical space $\mathbb{R}^3$ which is