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Advection-mediated competition in general environments

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Abstract

We consider a reaction–diffusion–advection system of two competing species with one of the species dispersing by random diffusion as well as a biased movement upward along resource gradient, while the other species by random diffusion only. It has been shown that, under some non-degeneracy conditions on the environment function, the two species always coexist when the advection is strong. In this paper, we show that for general smooth environment function, in contrast to what is known, there can be competitive exclusion when the advection is strong, and, we give a sharp criterion for coexistence that includes all previously considered cases. Moreover, when the domain is one-dimensional, we derive in the strong advection limit a system of two equations defined on different domains. Uniqueness of steady states of this non-standard system is obtained when one of the diffusion rates is large.

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1. Introduction

In this paper, we are interested in the effect of dispersal on the competition of species. Our study is motivated by an interesting result obtained in [11] in which Dockery, Hutson, Mischaikow and Pernarowski considered the following two species competition model

$$\begin{cases} U_t = d_1 \Delta U + U\big(m(x) - U - V\big) & \text{in } \Omega \times (0,\infty), \\ V_t = d_2 \Delta V + V\big(m(x) - U - V\big) & \text{in } \Omega \times (0,\infty), \\ \partial_\nu U = \partial_\nu V = 0 & \text{on } \partial\Omega \times (0,\infty), \\ U(x,0) = U_0(x), \quad V(x_0) = V_0(x) & \text{in } \Omega. \end{cases} \tag{1}$$

Here Ω is a bounded smooth domain in $\mathbb{R}^N$, with ν denoting the outward unit normal vector on the boundary $\partial\Omega$ of Ω, and $\partial_\nu = \nu \cdot \nabla$ being the outer normal derivative. U and V represent the population densities of two different competing species, while $m(x)$ captures the quality of the habitat Ω at location x. If $m(x)$ is nonconstant, it is shown that if $0 < d_2 < d_1$, then all positive solutions of (1), regardless of the initial values $U_0(x)$, $V_0(x)$, converge uniformly to $(0, \theta_{d_2})$ as $t \to \infty$, where θ_{d_2} is the unique positive steady state of (see, e.g. [4])

$$\begin{cases} \theta_t = d_2 \Delta \theta + \theta(m - \theta) & \text{in } \Omega \times (0,\infty), \\ \partial_\nu \theta = 0 & \text{on } \partial\Omega \times (0,\infty). \end{cases} \tag{2}$$

In other words, in pure diffusion models with heterogeneous environment, slower diffusion rate is favored.

In [20], an important distinction was drawn between unconditional dispersal, which does not depend on habitat quality or population density, and conditional dispersal, which does depend on such factors. Passive diffusion, as considered in [11], is an example of unconditional dispersal. Diffusion combined with directed movement upward along environmental gradients, as considered in [2,10], is a type of conditional dispersal.

As an attempt to determine whether conditional or unconditional dispersal strategy confers more ecological advantage, the following system was introduced in [5], following the approach in [11]:

$$\begin{cases} U_t = \nabla \cdot (d_1 \nabla U - \alpha U \nabla m) + U(m - U - V) & \text{in } \Omega \times (0,\infty), \\ V_t = d_2 \Delta V + V(m - U - V) & \text{in } \Omega \times (0,\infty), \\ d_1 \partial_\nu U - \alpha U \partial_\nu m = \partial_\nu V = 0 & \text{on } \partial\Omega \times (0,\infty). \end{cases} \tag{3}$$

While the two species U and V are ecologically equivalent, they adopt different dispersal strategies: V disperses purely randomly, and U adopts, in addition to diffusion, a directed movement upward along the environmental gradient ∇m. Throughout this paper, we always assume

(M1) $m \in C^2(\bar{\Omega})$ is nonconstant, and $\int_\Omega m > 0$.

Under assumption (M1), for all $d_i > 0$ and $\alpha \geq 0$, system (3) has a trivial steady state $(0,0)$, and two semi-trivial steady states $(\tilde{u}, 0)$ and $(0, \theta_{d_2})$, where θ_{d_2} is the unique positive