Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 257 (2014) 3501–3520
Journal of Differential Equations
www.elsevier.com/locate/jde

The rate at which energy decays in a viscously damped hinged Euler–Bernoulli beam

Kaïs Ammari [a,*], Mouez Dimassi [b], Maher Zerzeri [c]

[a] *UR Analyse et Contrôle des Edp, UR13ES64, Dép. de Mathématiques, Faculté des Sciences de Monastir, Université de Monastir, 5019 Monastir, Tunisia*

[b] *Université Bordeaux 1, CNRS, UMR 5251 IMB, 351, cours de la Libération, 33405 Talence cedex, France*

[c] *Université Paris XIII, CNRS, UMR 7539 LAGA, 99, ave Jean-Baptiste Clément, F-93430 Villetaneuse, France*

Received 20 June 2013; revised 12 March 2014

Available online 17 July 2014

Abstract

We study the best decay rate of the solutions of a damped Euler–Bernoulli beam equation with a homogeneous Dirichlet boundary conditions. We show that the fastest decay rate is given by the supremum of the real part of the spectrum of the infinitesimal generator of the underlying semigroup, if the damping coefficient is in $L^\infty(0, 1)$.

MSC: 74K10; 35Q72; 35B40; 34L20

Keywords: Rate of decay; Euler–Bernoulli beam; Spectral abscissa; Riesz basis

Contents

* Corresponding author.
E-mail addresses: kais.ammari@fsm.rnu.tn (K. Ammari), dimassi@math.u-bordeaux1.fr (M. Dimassi), zerzeri@math.univ-paris13.fr (M. Zerzeri).

http://dx.doi.org/10.1016/j.jde.2014.06.020

1. Introduction and main result

We consider the following damped Euler–Bernoulli equation:

$$\partial_t^2 u(x,t) + \partial_x^4 u(x,t) + 2a(x)\partial_t u(x,t) = 0, \quad 0 < x < 1,\ t > 0, \tag{1.1}$$

$$u(0,t) = u(1,t) = 0, \qquad \partial_x^2 u(0,t) = \partial_x^2 u(1,t) = 0, \quad t > 0, \tag{1.2}$$

$$u(x,0) = u^0(x), \qquad \partial_t u(x,0) = u^1(x), \quad 0 < x < 1, \tag{1.3}$$

where $a \in L^\infty(0,1)$ is non-negative satisfying the following condition:

$$\exists c > 0 \text{ s.t., } a(x) \geq c, \quad \text{a.e., in a non-empty open subset } I \text{ of } (0,1). \tag{1.4}$$

In order to formulate our results we consider the Hilbert space

$$\left[H^2(0,1) \cap H_0^1(0,1)\right] \times L^2(0,1) =: V \times L^2(0,1),$$

where we denote by $H^s(0,1)$, $s \in \mathbb{R}$, the usual Sobolev spaces. We endow this space with the inner product:

$$\left\langle [f,g],[u,v]\right\rangle := \int_0^1 \left(f^{(2)}(x)\overline{u^{(2)}(x)} + g(x)\overline{v(x)}\right)dx, \quad \text{for all } [f,g],[u,v] \text{ in } V \times L^2(0,1).$$

From now on, we shall represent a pair of functions by $[f,g]$ rather than (f,g) to avoid confusion with classical inner product on $L^2(0,1)$.

We define the energy of a solution u of (1.1)–(1.3), at time t, as

$$E\big(u(t)\big) = \frac{1}{2}\int_0^1 \left(\left|\partial_t u(x,t)\right|^2 + \left|\partial_x^2 u(x,t)\right|^2\right)dx. \tag{1.5}$$