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# The rate at which energy decays in a viscously damped hinged Euler–Bernoulli beam

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## Abstract

We study the best decay rate of the solutions of a damped Euler–Bernoulli beam equation with a homogeneous Dirichlet boundary conditions. We show that the fastest decay rate is given by the supremum of the real part of the spectrum of the infinitesimal generator of the underlying semigroup, if the damping coefficient is in  $L^\infty(0, 1)$ .

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## 1. Introduction and main result

We consider the following damped Euler–Bernoulli equation:

$$\partial_t^2 u(x, t) + \partial_x^4 u(x, t) + 2a(x)\partial_t u(x, t) = 0, \quad 0 < x < 1, \quad t > 0, \quad (1.1)$$

$$u(0, t) = u(1, t) = 0, \quad \partial_x^2 u(0, t) = \partial_x^2 u(1, t) = 0, \quad t > 0, \quad (1.2)$$

$$u(x, 0) = u^0(x), \quad \partial_t u(x, 0) = u^1(x), \quad 0 < x < 1, \quad (1.3)$$

where  $a \in L^\infty(0, 1)$  is non-negative satisfying the following condition:

$$\exists c > 0 \text{ s.t., } a(x) \geq c, \quad \text{a.e., in a non-empty open subset } I \text{ of } (0, 1). \quad (1.4)$$

In order to formulate our results we consider the Hilbert space

$$[H^2(0, 1) \cap H_0^1(0, 1)] \times L^2(0, 1) =: V \times L^2(0, 1),$$

where we denote by  $H^s(0, 1)$ ,  $s \in \mathbb{R}$ , the usual Sobolev spaces. We endow this space with the inner product:

$$\langle [f, g], [u, v] \rangle := \int_0^1 (f^{(2)}(x)\overline{u^{(2)}(x)} + g(x)\overline{v(x)}) dx, \quad \text{for all } [f, g], [u, v] \text{ in } V \times L^2(0, 1).$$

From now on, we shall represent a pair of functions by  $[f, g]$  rather than  $(f, g)$  to avoid confusion with classical inner product on  $L^2(0, 1)$ .

We define the energy of a solution  $u$  of (1.1)–(1.3), at time  $t$ , as

$$E(u(t)) = \frac{1}{2} \int_0^1 (|\partial_t u(x, t)|^2 + |\partial_x^2 u(x, t)|^2) dx. \quad (1.5)$$

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