Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations 258 (2015) 669–695

Journal of Differential Equations

www.elsevier.com/locate/jde

Traveling waves solutions of isothermal chemical systems with decay

Yuanwei Qi [a,b,*,1]

[a] *Department of Mathematics, University of Central Florida, Orlando, FL 32816, United States*
[b] *School of Mathematics, Shanxi University, Taiyuan, Shanxi, 030006, PR China*

Received 7 May 2014

Available online 11 November 2014

Abstract

This article studies propagating traveling waves in a class of reaction–diffusion systems which include a model of microbial growth and competition in a flow reactor proposed by Smith and Zhao [17], and isothermal autocatalytic systems in chemical reaction of order m with a decay order n, where m and n are positive integers and $m \neq n$. A typical system in autocatalysis is $A + 2B \to 3B$ (with rate $k_1 ab^2$) and $B \to C$ (with rate $k_2 b$), where $m = 2$ and $n = 1$, involving two chemical species, a reactant A and an auto-catalyst B whose diffusion coefficients, D_A and D_B, are unequal due to different molecular weights and/or sizes. Here a is the concentration density of A, b that of B and C an inert chemical species. The two constants k_1 and k_2 are material constants measuring the relative strength of respective reactions.

It is shown that there exist traveling waves when $m > 1$ and $n = 1$ with suitable relation between the ratio D_B/D_A, traveling speed c and rate constants k_1, k_2. On the other hand, it is proved that there exists no traveling wave when one of the chemical species is immobile, $D_B = 0$ or $n > m$ for all choices of other parameters.

MSC: 34C20; 34C25; 92E20

Keywords: Microbial growth in a flow reactor; Isothermal chemical systems with decay; Traveling wave; Existence; Non-existence

* Correspondence to: Department of Mathematics, University of Central Florida, Orlando, FL 32816, United States.
E-mail address: yuanwei.qi@ucf.edu.

[1] The author is partially supported by Shanxi Bairen plan of China. He thanks Yi Li and Junping Shi for stimulating discussions.

http://dx.doi.org/10.1016/j.jde.2014.09.013

1. Introduction

In this paper we study reaction–diffusion systems of the form

$$(\mathrm{I})\qquad \begin{cases} u_t = D_A u_{xx} - f(u,v), \\ v_t = D_B v_{xx} + f(u,v) - g(v), \end{cases} \tag{1.1}$$

where f is a C^1 function defined on $[0,\infty)\times[0,\infty)$, g a C^1 function defined on $[0,\infty)$ with properties

$$f(u,0) = f(0,v) = 0, \quad \text{and} \quad f(u,v) > 0 \quad \text{on } (0,\infty)\times(0,\infty),$$
$$g(0) = 0 \quad \text{and} \quad g(v) > 0 \quad \text{on } (0,\infty),$$

where D_A, D_B are positive constants representing the diffusion coefficients of two different species. The particular feature we are interested in is the existence and non-existence of traveling waves. Without loss of generality, we shall assume in what follows that $D_A = 1$ and use d in place of D_B, since the general case can be transformed to this one by a simple non-dimensional scaling.

Many interesting phenomena in population dynamics, bio-reactors and chemical reactions can be modeled by a system of the form as in (1.1). For example, a system modeling microbial growth and competition in a flow reactor was first studied in [1] and [17], where a special case is $f(u,v) = F(u)v$, $g(v) = Kv$, K a positive constant, and $F(0) = 0$ and $F'(0) > 0$. In that context, u is the density of nutrient and v that of microbial population. $g(v)$ is the death rate of microbial. Subsequent works with emphasis on traveling waves appeared later in [18] and more recently in [10]. Furthermore, when $F(u) = u$, it is reduced to a classical diffusive epidemic model of Kermack and Mckendric [11].

Another interesting case arises from isothermal autocatalytic chemical reaction between two chemical spices A and B taking the form:

$$A + mB \longrightarrow (m+1)B \quad \text{with rate } r[A][B]^m,$$

where m is an integer and $r > 0$ is a rate constant. In that situation, $f(u,v) = uv^m$ with u the concentration density of A and v that of B. If there is no decay, then $g(v) = 0$. The resulting system is

$$\begin{cases} u_t = u_{xx} - uv^m, \\ v_t = dv_{xx} + uv^m, \end{cases} \tag{1.2}$$

after a simple non-dimensional transformation. The global dynamics of the Cauchy problem as well as existence of traveling wave, sharp estimate of minimum speed and stability were investigated in [3,4,12,13,15] for $m > 1$ case.

Furthermore, it was demonstrated in [2] by asymptotic analysis and numerical computation, and rigorously proved for $m = 1$ in [5] that any small amount of B introduced locally with uniform initial distribution of A can generate traveling wave. The feature seems to be contradictory to the fact that in the relevant experimental result of chemical reactions, the initiation of traveling wave calls for sufficient amount of B to be added [20]. To overcome this lacking of threshold