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Bifurcation diagrams for Hamiltonian linear type centers of linear plus cubic homogeneous polynomial vector fields

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Abstract

As a natural continuation of the work done in [7] we provide the bifurcation diagrams for the global phase portraits in the Poincaré disk of all the Hamiltonian linear type centers of linear plus cubic homogeneous planar polynomial vector fields.

MSC: 34C07; 34C08

Keywords: Hamiltonian linear type center; Cubic polynomial; Vector fields; Phase portrait; Bifurcation diagram

1. Introduction and statement of the main results

Limit cycles and, being closely related, the center-focus problem have been among the main subjects that recently attracted a lot of attention in the qualitative theory of real planar differential systems. The center-focus problem refers to determining whether a singular point is a center or

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a focus. The definition of center was first introduced by Poincaré in [18]. He defined a center as a singular point of a vector field on the real plane which has a neighborhood that consists solely of periodic orbits and the singular point itself.

Analytic differential systems having a center at the origin are grouped in three categories. If after an affine change of variables and a rescaling of the time variable the differential system can be written in the form

$$\dot{x} = -y + P(x, y), \qquad \dot{y} = x + Q(x, y),$$

then it is called a *linear type center*; if it can be written in the form

$$\dot{x} = y + P(x, y), \qquad \dot{y} = Q(x, y),$$

then it is called a *nilpotent center*; and finally if it can be written in the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y),$$

then it is called a *degenerate center*. Here $P(x, y)$ and $Q(x, y)$ are real analytic functions without constant and linear terms, defined in a neighborhood of the origin. For the characterization of linear type centers Poincaré [19] and Lyapunov [15] provide an algorithm; we also refer to Chazy [5] and Moussu [17]. On the other hand an algorithm for the characterization of nilpotent and some class of degenerate centers are given by Chavarriga et al. [4], Cima and Llibre [6], Giacomini et al. [11], and Giné and Llibre [12].

The classification of centers of quadratic polynomial differential systems started with the works of Dulac [9], Kapteyn [13,14] and Bautin [1]. In [22] Vulpe provides all the global phase portraits of quadratic polynomial differential systems having a center. Schlomiuk [20] and Żołądek [25] provided the bifurcation diagrams of all quadratic differential systems having a center.

Considering the classification of the centers of polynomial differential systems with degrees higher than two there are many but partial results. For linear type centers of cubic polynomial differential systems having linear terms with homogeneous nonlinearities of degree three were characterized by Malkin [16], and by Vulpe and Sibirski [23]. We provide all the global phase portraits of Hamiltonian linear type and nilpotent centers of linear plus cubic homogeneous polynomial vector fields in [7] and [8], respectively. In addition we refer to Rousseau and Schlomiuk [21], and Żołądek [26,27] for some interesting results in some subclasses of cubic systems. Systems with homogeneous nonlinearities of higher degree having linear type centers are not fully characterized, but see Chavarriga and Giné [2,3] for some of the main results. In any case there is still a long way to fully characterize and classify the centers of all polynomial differential systems of degree three.

In this work we provide the bifurcation diagrams for the global phase portraits in the Poincaré disk of all the Hamiltonian linear type centers of linear plus cubic homogeneous planar polynomial vector fields. We say that two vector fields on the Poincaré disk are *topologically equivalent* if there exists a homeomorphism from one onto the other which sends orbits to orbits preserving or reversing the direction of the flow. In [8] the global phase portraits of all Hamiltonian planar polynomial vector fields with only linear and cubic homogeneous terms having a linear center at the origin are given by the following theorem: