Large time behavior of a solution of carbon dioxide transport model in concrete carbonation process

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Abstract

In [7] we show the global existence and uniqueness of a solution of carbon dioxide transport model in concrete carbonation process. This model is governed by a parabolic-type equation which has a non-local term depending on the unknown function itself. In this paper, we show the large time behavior of that solution.

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1. Introduction

In this paper we consider the following initial boundary value problem (P) = {(1.1), (1.2), (1.3)}, which is a mathematical model of carbon dioxide transport in concrete carbonation process:

$$\frac{\partial}{\partial t}\left[\phi\left(1 - e^{-\int_0^t u(\tau)d\tau}\right)u\right] - \Delta u = -w_0 u e^{-\int_0^t u(\tau)d\tau} \quad \text{in } Q(T) := (0, T) \times \Omega, \quad (1.1)$$

$$u = u_b \quad \text{on } S(T) := (0, T) \times \Gamma, \quad (1.2)$$

$$u(0) = u_0 \quad \text{in } \Omega. \quad (1.3)$$
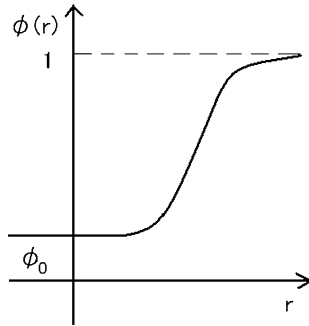
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Fig. 1. The graph of ϕ.

Here, Ω is a bounded domain of R^3 with a smooth boundary $\Gamma = \partial\Omega$, $T > 0$ is a fixed finite number, ϕ is a function in $\mathrm{C}^1(\mathrm{R})$ satisfying $\phi_0 \le \phi(r) \le 1$ for $r \in \mathrm{R}$ where ϕ_0 is a positive constant, u_b is a given function on $Q(T)$, and w_0 and u_0 also are given functions on Ω.

Eq. (1.1) is a diffusion equation derived from the following balanced law of carbon dioxide in concrete carbonation process proposed by Maekawa, Chaube and Kishi [9] and Maekawa, Ishida and Kishi [10]:

$$\frac{\partial}{\partial t}\big\{\phi\big[(1-\mathcal{S})v + \mathcal{S}u\big]\big\} - \mathrm{div}\big(\phi\big[\mathcal{H}_1(1-\mathcal{S})\nabla v + \mathcal{H}_2\mathcal{S}\nabla u\big]\big) = -\kappa u w \quad \text{in } Q(T). \quad (1.4)$$

Here, we explain briefly the physical background to (1.4) (see [2] for details). In (1.4), Ω is a domain occupied by concrete, and the unknown functions $v = v(t, x)$ and $u = u(t, x)$ represent the concentrations of carbon dioxide in air and in water at a time t and a position $x \in \Omega$, respectively. In the equilibrium state, the relation $v = \rho_0 u$ is known to hold for a positive constant ρ_0 by Henry's law. Also, $\phi = \phi(z)$ represents the porosity, which is the ratio of the volume of the voids inside of the concrete to the volume of the whole concrete (see Fig. 1), and z is the ratio of the volume of consumed calcium hydroxide to the volume of the total calcium hydroxide. Next, $\mathcal{S}$ represents the degree of saturation corresponding to the relative humidity; its relationship to the degree of saturation is given as a hysteresis operator in [9,10]. Accordingly, $\phi(1 - \mathcal{S})v$ and $\phi\mathcal{S}u$ represent the concentration of gaseous carbon dioxide in pores and the concentration of carbon dioxide dissolved in pore water, respectively.

In the diffusion term, $\mathcal{H}_1$ and $\mathcal{H}_2$ are positive constants, and $\mathcal{H}_1(1 - \mathcal{S})$ and $\mathcal{H}_2\mathcal{S}$ represent the diffusion coefficients of gaseous carbon dioxide in pore and carbon dioxide dissolved in water. In the forcing term, κ is a reaction rate and w represents the concentration of calcium ion. The forcing term represents the consumed carbon dioxide during the carbonation of concrete, and is given by the reaction rate theory.

Eq. (1.1) is obtained from (1.4) by the following procedure. First, we use the facts that w and z (for z, see below (1.4)) are expressible in the following form:

$$w(t) = \big[\mathrm{Ca}^{2+}\big](0)e^{-\tilde{\kappa}\int_0^t u(\tau)d\tau}u(t), \qquad z(t) = 1 - e^{-\tilde{\kappa}\int_0^t u(\tau)d\tau} \quad \text{for } t > 0. \quad (1.5)$$

Here, $\tilde{\kappa}$ is a positive constant, $[\mathrm{Ca}^{2+}](0)$ is the concentration of calcium ion at an initial time, and u is the concentration of carbon dioxide in water, which is the unknown function. Henceafter, [a] represents the concentration of the element a. As mentioned above, the forcing term in (1.4) is given by the following form based on the reaction rate theory: