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Asymptotic profiles for wave equations with strong damping

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Abstract

We consider the Cauchy problem in $\mathbf{R}^n$ for strongly damped wave equations. We derive asymptotic profiles of these solutions with weighted $L^{1,1}(\mathbf{R}^n)$ data by using a method introduced in [9] and/or [10].

MSC: primary 35B40, 35L15; secondary 35L05, 35C20

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1. Introduction

We are concerned with the Cauchy problem for strongly damped wave equations in $\mathbf{R}^n$ ($n \geq 1$):

$$u_{tt}(t,x) - \Delta u(t,x) - \Delta u_t(t,x) = 0, \quad (t,x) \in (0,\infty) \times \mathbf{R}^n, \tag{1.1}$$

$$u(0,x) = u_0(x), \qquad u_t(0,x) = u_1(x), \quad x \in \mathbf{R}^n, \tag{1.2}$$

where the initial data u_0 and u_1 are taken from the energy space:

$$[u_0, u_1] \in H^1(\mathbf{R}^n) \times L^2(\mathbf{R}^n).$$

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It is known (see [13]) that the problem (1.1)–(1.2) admits a unique weak solution $u \in C([0,+\infty); H^1(\mathbf{R}^n)) \cap C^1([0,+\infty); L^2(\mathbf{R}^n))$.

The purpose of this paper is to investigate the asymptotic profiles of the solution $u(t,x)$ to problem (1.1)–(1.2). As for the related results Ponce [20] and Shibata [22] have already studied the L^p–L^q decay estimates of solutions to problem (1.1)–(1.2). The exterior domain case for Eq. (1.1) was also handled in Ikehata [7], in which the two dimensional case is likely to be sharp about the rate of decay of the corresponding total energy and the L^2-norm of solutions. Recently Ikehata–Natsume [11] derived the decay estimates of the total energy and L^2-norm of solutions to (1.1)–(1.2) with a more general structural damping based on the energy method in the Fourier space due to [24]. Before [11], Ikehata–Todorova–Yordanov [13] succeeded to find the asymptotic profile in the abstract framework, in fact, they studied the following OD equations in Hilbert space H:

$$u_{tt}(t) + Au(t) + Au_t = 0, \quad u(0) = u_0, \quad u_t(0) = u_1, \tag{1.3}$$

where A is a nonnegative self-adjoint operator in H. They employed the abstract energy method in the Fourier space combined with the spectral analysis to find the asymptotic profile such as

$$u(t) \sim e^{-tA/2}\big(\cos\big(tA^{1/2}\big)u_0 + A^{-1/2}\sin\big(tA^{1/2}\big)u_1\big), \quad (t \to +\infty).$$

These ideas are inspired from [2] and [24]. Therefore, in this sense we have already caught the asymptotic profiles from the work [13], however, it seems to be important to search another root to find the profiles of the solution to (1.1)–(1.2) by the concrete way because one can sometimes find a possibility of several new applications of the method introduced in this paper. Our new point of view is in dealing with the problem (1.1)–(1.2) in a framework of the weighted L^1-data. By imposing some weights on the initial data in L^1 sense we can get the meaningful “equality” represented by (2.11) below, which includes explicitly the leading plus remainder terms. This method is basically independent from the shape of equation itself. The origin of this idea comes from [8,9], which studied the decay property of solutions to the damped wave equations:

$$u_{tt}(t,x) - \Delta u(t,x) + u_t(t,x) = 0. \tag{1.4}$$

The research in the framework of the weighted L^1-data was also developed more precisely to (1.4) in the recent results due to [15] and [10]. Especially [15] dealt with the nonlinear problems of (1.4). The decay property and the asymptotic profiles to Eq. (1.4) are well-studied in [5,10,12, 14,16,18,19,21]. As compared with Eq. (1.4), there seems to be few results about the asymptotic profiles of solutions to Eq. (1.1), so it is good chance to present a way to investigate the asymptotic behavior of solutions to (1.1)–(1.2). In this connection, quite recently vigorous works about the global existence of solutions and/or a new method to derive sharper decay estimates of the total energy to the Cauchy problem of the equation

$$u_{tt} - \Delta u + (-\Delta)^\sigma u_t = \mu f(u), \quad \sigma \in [0,1]$$

are successively announced by D’Abbicco–Reissig [3] ($\mu > 0$) and Charão–daLuz–Ikehata [1] ($\mu = 0$), respectively.