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A periodic reaction–advection–diffusion model for a stream population ☆

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Abstract

This paper is devoted to the study of spatial dynamics of a periodic reaction–advection–diffusion model for a stream population. In the case of an unbounded domain, we establish the existence of the leftward and rightward spreading speeds and their coincidence with the minimal wave speeds for monotone periodic traveling waves, respectively. In the case of a bounded domain, we obtain a threshold result on the global stability of either zero or the positive periodic solution.

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Keywords: Reaction–advection–diffusion model; Spreading speeds; Periodic traveling waves; Positive periodic solution; Global stability

1. Introduction

In stream ecology, 'drift paradox' [11] is a crucial topic, which is concerned about why the aquatic insects have the ability to resist washout when they face with the downstream drift. A number of modeling works have been done to give positive answers for the drift paradox. Pachepsky et al. [13] introduced a reaction–advection–diffusion model (called PA model) to handle the issue of persistence of benthic aquatic organisms. In this PA model, the population

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is divided into two interacting compartments: individuals living on the benthos and individuals drifting in the river, which has important implications for population persistence. Later, Lustscher and Seo [10] further developed this PA model by considering the temporal variability, and analytically studied the persistence conditions for the linearized PA model under the assumption that all parameters are T-periodic step functions and the average flow speed over one period is constant. Their model is governed by the following linear reaction–advection–diffusion system:

$$\begin{cases} \dfrac{\partial n_d}{\partial t} = -\sigma(t)n_d + \mu(t)n_b - v(t)\dfrac{\partial n_d}{\partial x} + D(t)\dfrac{\partial^2 n_d}{\partial x^2}, \\ \dfrac{\partial n_b}{\partial t} = \sigma(t)n_d - \mu(t)n_b + r(t)n_b, \quad t > 0,\ x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where n_d is the population density in the drift; n_b is the population density on the benthos; $\mu(t)$ is the per capita rate at which individuals in the benthic population enter the drift; $\sigma(t)$ is the per capita rate at which the organisms return to the benthic population from drifting; $D(t)$ is the diffusion coefficient; $v(t)$ is the advection speed experienced by the organisms; and $r(t)$ is the maximum per capita growth rate of the benthic population. The purpose of the current paper is to study the spatial dynamics of the following nonlinear periodic PA model:

$$\begin{cases} \dfrac{\partial n_d}{\partial t} = -\sigma(t)n_d + \mu(t)n_b - v(t)\dfrac{\partial n_d}{\partial x} + D(t)\dfrac{\partial^2 n_d}{\partial x^2}, \\ \dfrac{\partial n_b}{\partial t} = \sigma(t)n_d - \mu(t)n_b + f(t, n_b)n_b, \quad t > 0,\ x \in \mathbb{R}, \end{cases} \tag{1.2}$$

where $f(t, n_b)$ is the per capita growth rate of the benthic population with no Allee effect in the population. Biologically, model (1.2) is more reasonable since it deals with seasonal variations in temperature, rainfall and resource availability.

Note that system (1.2) is cooperative and its solution maps are monotone. Thus, we can use the general theory developed in [16,8] to study the spreading speeds for periodic system (1.2). However, the solution maps are not compact with respect to the compact open topology due to the lack of the diffusion term in the second equation of system (1.2). As a consequence, the theory in [16,8] may not be applied to obtain the existence of time-periodic traveling waves for system (1.2). To overcome this difficulty, we will utilize the theory recently developed in [2] for monotone semiflows with weak compactness. We should point out that the verification of some abstract assumptions in [2] is highly nontrivial for the solution maps of system (1.2) since one needs to consider its mild solutions with discontinuous initial functions. It turns out that the spreading speeds are linearly determinate and coincide with the minimum wave speeds for monotone periodic traveling waves. For the global dynamics of system (1.2) in a bounded domain, we will appeal to the theory of monotone and subhomogeneous systems (see, e.g., [18]). To avoid using the compactness for solution maps, we prove that every forward orbit of the Poincaré map associated with system (1.2) is asymptotically compact under an additional assumption, and that the omega limit set of every nonzero forward orbit is contained in the interior of the positive cone in the case where the zero solution is linearly unstable. This enables us to establish a threshold type result on the global stability of either zero or the positive periodic solution.

The remaining part of the paper is organized as follows. In Section 2, we first obtain a threshold dynamics for the spatially homogeneous system of model (1.2) in terms of the principal