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Periodic solutions of superlinear impulsive differential equations: A geometric approach

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Abstract

A geometric method is introduced to study superlinear second order differential equations with impulsive effects. Basing on a reference continuous polar lifting of a planar orientation-preserving homeomorphism, we prove via the Poincaré–Birkhoff twist theorem the existence of infinitely many periodic solutions of conservative superlinear second order equations with the finite twist impulsive terms. We also prove, by developing a new twist fixed point theorem, the existence of periodic solutions for non-conservative superlinear second order equations with degenerate impulsive terms.

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1. Introduction

Consider a superlinear second order differential equation

$$x'' + g(x) = p(t, x, x'), \tag{1.1}$$

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where $g : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying the superlinear growth condition

$$(g_0) \qquad \lim_{|x|\to\infty} g(x)/x = +\infty$$

and $p : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is bounded, continuous and 2π-periodic in the first variable.

Superlinear second order differential equation is one of the typical models both in ODE and forced vibrations. There are many interesting results on the existence and multiplicity of periodic solutions of superlinear second order differential equations. The used methods range from the Poincaré–Birkhoff twist theorem [9,13,14], variational method [1,2,18] to Leray–Schauder continuation method of topological degree [4,5].

In case Eq. (1.1) is conservative, the Poincaré map of (1.1) is an area-preserving homeomorphism. Following a suggestion by Moser, Jacobowitz [14] first applied the celebrated Poincaré–Birkhoff twist theorem to prove the existence of infinitely many periodic solutions for superlinear second order equations. One can also refer to the work of Hartman [13]. Later, Ding and Zanolin [9] applied a generalized version of the Poincaré–Birkhoff theorem by Ding [8] to prove the existence of infinitely many periodic solutions for the Duffing equations with superquadratic potential. We also refer to [11,25,26,28,30] and the references therein for the related researches.

In case Eq. (1.1) is nonconservative, Capietto, Mawhin and Zanolin [4,5] developed a continuation theorem to prove the existence of periodic solutions of superlinear second order differential equations. If $p = 0$, then an elementary study of (1.1) under superlinear condition (g_0), based upon the first integral of energy, reveals that (1.1) has infinitely many 2π-periodic solutions with arbitrary large amplitudes. So usually the set of possible periodic solutions of a family of equations connecting superlinear equation (1.1) is not a priori bounded. In the Leray–Schauder continuation method a priori estimate is used to guarantee that the corresponding topological degree is not zero. The success of the continuation theorem by Capietto, Mawhin and Zanolin is based upon the analysis of the rapid rotation property of solutions in phase-plane. The rapid rotation property is usually caused by the superlinear growth of g and corresponds to the so-called twist property of the Poincaré map for (1.1).

We are concerned in this paper with the existence and multiplicity of periodic solutions for superlinear impulsive second order differential equations of the form

$$\begin{cases} x'' + g(x) = p\big(t, x, x'\big), & t \neq t_j; \\ \Delta x(t_j) = I_j\big(x(t_j-), x'(t_j-)\big), & \\ \Delta x'(t_j) = J_j\big(x(t_j-), x'(t_j-)\big), & j = \pm 1, \pm 2, \cdots, \end{cases} \tag{1.2}$$

where $0 \le t_1 < \cdots < t_k < 2\pi$, $\Delta x(t_j) = x(t_j+) - x(t_j-)$, $\Delta x'(t_j) = x'(t_j+) - x'(t_j-)$, and $I_j, J_j : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are continuous maps for $j = \pm 1, \pm 2, \cdots$. In addition, assume that the impulsive time is 2π-periodic, that is, $t_{j+k} = t_j + 2\pi$ and $I_{j+k} = I_j$, $J_{j+k} = J_j$ for $j = \pm 1, \pm 2, \cdots$.

Impulsive differential equations serve as basic models in the study of the dynamics of processes subject to sudden changes in their states. We refer the reader to classical monographs [3,16,24] for the general aspects of impulsive differential equations, [20–22] for the existence of periodic solutions of impulsive differential equations via fixed point theory, [10,27] via topological degree theory, and [23,31,32] via variational method.

However, different from the extensive study for superlinear second order differential equation without impulsive terms, there is only a few results on the existence and multiplicity of periodic solutions for superlinear impulsive second order differential equations.