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Using Lin's method to solve Bykov's problems

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Abstract

We consider nonwandering dynamics near heteroclinic cycles between two hyperbolic equilibria. The constituting heteroclinic connections are assumed to be such that one of them is transverse and isolated. Such heteroclinic cycles are associated with the termination of a branch of homoclinic solutions, and called *T-points* in this context. We study codimension-two T-points and their unfoldings in $\mathbb{R}^n$. In our consideration we distinguish between cases with real and complex leading eigenvalues of the equilibria. In doing so we establish Lin's method as a unified approach to (re)gain and extend results of Bykov's seminal studies and related works. To a large extent our approach reduces the study to the discussion of intersections of lines and spirals in the plane.

Contents

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1. Introduction

Homoclinic bifurcations lie at the heart of our understanding of complicated (chaotic) recurrent behaviour in dynamical systems. The history goes back to Poincaré, with major subsequent contributions by the schools of Andronov, Shilnikov, Smale and Palis. The success of the latter schools has been founded on a combination of analytical and geometrical tools; typical for the field of dynamical systems.

We consider a parameter family of vector fields $f : \mathbb{R}^n \times \mathbb{R}^2 \to \mathbb{R}^n$ $(n \geq 3)$, f smooth:

$$\dot{x} = f(x, \mu). \tag{1.1}$$

We study the nonwandering dynamics in the neighbourhood of a *T-point* [1,36,20]: a heteroclinic cycle between two hyperbolic equilibria of saddle type p_1 and p_2, where one of the connections is transverse and isolated. See Fig. 1 for a sketch of a T-point heteroclinic cycle in $\mathbb{R}^3$. T-points have been found to appear in many applications of interest, ranging from the Lorenz [20] and Kuramoto–Sivashinsky [32] systems, to electronic oscillators [2,3,15–17], semiconductor lasers [21,46], magnetoconvection [40] and travelling waves in reaction–diffusion dynamics [29,44,39,23].