

Spreading of the free boundary of an ideal fluid in a vacuum

Thomas C. Sideris

Department of Mathematics, University of California, Santa Barbara, CA 93106, USA

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Abstract

The diameter of a region occupied by an ideal fluid surrounded by vacuum will be shown to grow linearly in time provided the pressure is positive and there are no singularities. A family of explicit spherically symmetric, self-similar global solutions is constructed which illustrate the result in the compressible case.

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1. Introduction

Consider an ideal fluid occupying a bounded region in space surrounded by vacuum. The boundary of this region is free to move with the fluid. We shall show under the physically reasonable assumptions of positive density and pressure that the diameter of this region grows at least linearly in time as long as the fluid motion remains C^1 . The linear spreading rate is illustrated with an explicit family of global spherically symmetric, self-similar compressible solutions.

The method of proof is based on simple identities for integral averages as introduced in [1] and further explored in [2–4]. In the earlier works [1–3], the goal was to establish formation of singularities in classical solutions in the entire space, but the relationship between regularity and propagation of wave fronts plays a major role. For classical solutions in the entire space, the maximum propagation speed of a front into a constant state is *a priori* determined by the background state, whereas in the case of a free boundary, the speed at which the disturbance penetrates the vacuum will, in general, depend on the unknown solution. Although there is a certain amount of flexibility, the method will be applied in two situations: compressible ideal gasses and incompressible ideal fluids, see [Theorems 1 and 2](#).

The local existence of classical solutions to the initial free boundary value problem for ideal fluids has been much-studied in the last decade. The full water wave problem for incompressible, irrotational flow with gravity was considered in 2d and 3d, see [5] and [6], respectively. Estimates for the incompressible case without gravity were initiated in [7], well-posedness for the linearized problem was established in [8], and well-posedness for the nonlinear problem was subsequently proven in [9], see also [10,11]. The well-posedness for compressible liquids (positive fluid density on the free boundary) was established in [12], see also [13,14]. Finally, local existence for compressible gasses (vanishing density on the free boundary) was recently shown in [15,16], see also [17]. A key role in these works is played by a vacuum boundary condition. For liquids, the normal derivative of the pressure is negative on the free boundary. This implies that the pressure is positive near the boundary, which is consistent with condition (6) in Theorem 2. For compressible fluids, the normal derivative of the square of the sound speed must be negative. This is consistent with our result in Theorem 1, as well as with the example presented in Theorem 3.

1.1. General problem

Suppose that for times $0 \leq t \leq T$ a three-dimensional fluid occupies a bounded open region Ω_t , with C^1 boundary $\partial\Omega_t$. Define the space–time region

$$\mathcal{C}_T = \{(t, x): x \in \Omega_t, 0 < t < T\},$$

and assume that its lateral (free) boundary

$$\mathcal{B}_T = \{(t, x): x \in \partial\Omega_t, 0 \leq t < T\}$$

is C^1 with unit outward normal $\eta(t, x) \in \mathbb{R}^4$, for $(t, x) \in \mathcal{B}_T$.

The motion of an ideal fluid is modeled by the Euler equations involving the density ρ , velocity u , and the pressure p . These are

$$D_t \rho + \rho \nabla \cdot u = 0, \quad \text{in } \mathcal{C}_T, \quad (1a)$$

$$\rho D_t u + \nabla p = 0, \quad \text{in } \mathcal{C}_T, \quad (1b)$$

where $D_t = \partial_t + u \cdot \nabla$ is the usual material time derivative.

Assume that Ω_t moves with the fluid flow, so that

$$(1, u) \cdot \eta = 0, \quad \text{on } \mathcal{B}_T. \quad (2a)$$

We assume that the fluid is surrounded by vacuum, whence the normal stress should also vanish on the free boundary. In the case of fluids, the appropriate boundary condition is

$$p = 0, \quad \text{on } \mathcal{B}_T. \quad (2b)$$

Of course, the system (1a), (1b) is underdetermined, and further assumptions are necessary. We shall consider two cases: ideal gasses and incompressible ideal fluids.

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