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The spreading front of invasive species in favorable habitat or unfavorable habitat ☆

Chengxia Lei [a], Zhigui Lin [b,*], Qunying Zhang [b]

[a] *Department of Mathematics, University of Science and Technology of China, Hefei 230026, China*
[b] *School of Mathematical Science, Yangzhou University, Yangzhou 225002, China*

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Abstract

Spatial heterogeneity and habitat characteristic are shown to determine the asymptotic profile of the solution to a reaction–diffusion model with free boundary, which describes the moving front of the invasive species. A threshold value $R_0^{Fr}(D,t)$ is introduced to determine the spreading and vanishing of the invasive species. We prove that if $R_0^{Fr}(D,t_0) \geqslant 1$ for some $t_0 \geqslant 0$, the spreading must happen; while if $R_0^{Fr}(D,0) < 1$, the spreading is also possible. Our results show that the species in the favorable habitat can establish itself if the diffusion is slow or the occupying habitat is large. In an unfavorable habitat, the species dies out if the initial value of the species is small. However, big initial number of the species is benefit for the species to survive. When the species spreads in the whole habitat, the asymptotic spreading speed is given. Some implications of these theoretical results are also discussed.

MSC: primary 35R35; secondary 35K60

Keywords: Reaction–diffusion systems; Logistic model; Free boundary; Favorable habitat; Spreading

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* Corresponding author.
E-mail address: zglin68@hotmail.com (Z.G. Lin).

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1. Introduction

There have been some recent studies on reaction–diffusion models to understand the nature of spreading of the invasive species. The spreading of species from their native habitats to alien environments is a serious threat to biological diversity [25]. Subsequently, many mathematical models have been constructed to investigate how raw species survive in the habitat [30]. Among those models, there was a well-known model, which is described by the diffusive logistic equation over the entire space $\mathbb{R}^n$:

$$u_t - d\Delta u = u(a - bu), \quad x \in \mathbb{R}^n, \ t > 0, \tag{1.1}$$

where $u = u(x,t)$ is the population density of an invasive species with diffusion rate d, intrinsic growth rate a and habitat carrying capacity a/b. For space dimension $n = 1$, traveling wave solutions have been found by Fisher [13] and Kolmogorov et al. [18]. That is, for any $c \geqslant c^* := 2\sqrt{ad}$, there exists a solution $u(x,t) := W(x - ct)$ with the property that

$$W'(y) < 0, \quad y \in \mathbb{R}^1, \qquad W(-\infty) = a/b, \qquad W(\infty) = 0;$$

no such solution exists if $c < c^*$. The number c^* is then called the minimal speed of the traveling waves. The related research and recent developments can be found, for example, in [8,16] and the references therein.

However, the solution to problem (1.1) with any nontrivial initial population $u(x,0)$ is always positive everywhere, which means that any invasive species can establish itself in any new environment. This contrasts sharply with numerous empirical evidences, for example, let us see a biological control programme for broom began in New Zealand in 1981. A field experiment was used to manipulate the critical first stages of the invasion of the psyllid, Arytainilla spartiophila. Fifty-five releases were made along a linear transect 135 km long, six years after their original release, psyllids were present in 22 of the 55 release sites [2]. The field experiment showed that not all releases of psyllids survived and successful establishment is a complex process.

To describe precisely the spreading front of invasive species, Du and Lin [7] studied the following free boundary problem,

$$\begin{cases} u_t - du_{xx} = u(a - bu), & 0 < x < h(t), \ t > 0, \\ u_x(0,t) = u\big(h(t),t\big) = 0, & t > 0, \\ h'(t) = -\mu u_x\big(h(t),t\big), & t > 0, \\ u(x,0) = u_0(x), & 0 \leqslant x \leqslant h_0, \end{cases} \tag{1.2}$$

where $x = h(t)$ is the free boundary to be determined, d, a, h_0, μ and b are given positive constants, the unknown $u(x,t)$ stands for the population density of an invasive species over a one-dimensional habitat, and the initial function $u_0(x)$ stands for the population of the species in the early stage of its introduction. It is assumed that the spreading front expands at a speed that is proportional to the population gradient at the front, which gives rise to the classical Stefan condition $h'(t) = -\mu u_x(h(t),t)$, the positive constant μ measures the ability of the invasive species to transmit and diffuse in the new habitat, see [21] in details.

A spreading–vanishing dichotomy was first presented in [7] for problem (1.2), namely, as time approaches to infinity, the population $u(x,t)$ either successfully establishes itself in the new environment (called spreading), in the sense that $h(t) \to \infty$ and $u(x,t) \to a/b$, or the population