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Global Hopf bifurcation of differential equations with threshold type state-dependent delay

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Abstract

We develop global Hopf bifurcation theory of differential equations with state-dependent delay using the S^1-equivariant degree and investigate a two-degree-of-freedom mechanical model of turning processes. For the model of turning processes we show that the extreme points of each vibration component of the non-constant periodic solutions can be embedded into a manifold with explicit algebraic expression. This observation enables us to establish analytical upper and lower bounds of the amplitudes of the periodic solutions in terms of the system parameters and to exclude certain periods. Using the achieved global bifurcation theory we reveal that if the relative frequency between the natural frequency and the turning frequency varies in a certain interval, then generically every bifurcated continuum of periodic solutions must terminate at a bifurcation point. This termination means that the underlying system with parameters in the stability region near the vertical asymptotes of the stability lobes is less subject to chatter. In the process, several sufficient conditions for the non-existence of non-constant periodic solutions are also obtained.

Keywords: State-dependent delay; Turning processes; Global Hopf bifurcation; A priori bounds; S^1-equivariant degree

Contents

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1. Introduction

Machine-tool chatter is a self-excitation phenomenon occurring in cutting a metal workpiece, when the cutting process tends to decrease the machine structural damping ending with violent vibrations, uneven workpiece finish and possibly damage to the machining tool. Extensive efforts till recent years have contributed to the understanding of the mechanism of chatter, the suppression of vibrations and numerical simulations. See, e.g., [12,13,16,17,21,23,25,26,28,29,34–36]. A mathematical model for the turning process was developed in [16] which can be described as follows. Consider a turning process as shown in Fig. 1. The tool is assumed to be compliant and has bending oscillations in orthogonal directions x and y. The governing equations read

$$m\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) = F_x, \tag{1.1}$$

$$m\ddot{y}(t) + c_y\dot{y}(t) + k_y y(t) = -F_y. \tag{1.2}$$

The x and y components of the cutting process force can be written as

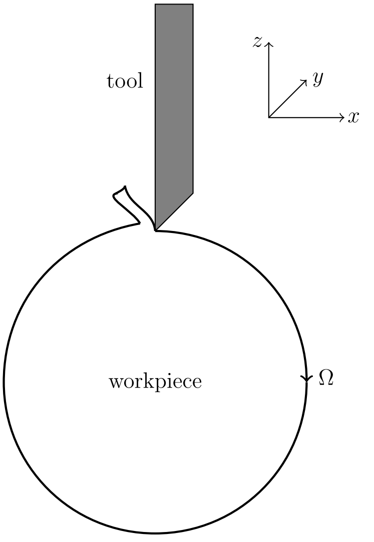

Fig. 1. Turning model.