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Parabolic equations with exponential nonlinearity and measure data

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Abstract

Let Ω be a bounded domain in $\mathbb{R}^N$ and $T > 0$. We study the problem

$$(P_\pm) \qquad \begin{cases} u_t - \Delta u \pm g(u) = \mu & \text{in } Q_T := \Omega \times (0,T) \\ u = 0 & \text{on } \partial\Omega \times (0,T) \\ u(.,0) = \omega & \text{in } \Omega \end{cases}$$

where μ and ω are bounded measures in Q_T and Ω respectively and $g(u) \sim e^{a|u|^q}$ with $a > 0$ and $q \geq 1$. We provide a sufficient condition in terms of fractional maximal potentials of μ and ω for solving $(P_\pm)$. Moreover, we prove uniqueness for (P_+).

MSC: 35K15; 35K58; 35R06

Keywords: Semilinear parabolic equations; Exponential nonlinearity; Parabolic Wolff potential; Radon measures

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1. Introduction

1.1. Introduction of problem and a brief survey of literature

In this paper, we study the following problem

$$\begin{cases} u_t - \Delta u \pm g(u) = \mu & \text{in } Q_T := \Omega \times (0,T) \\ u = 0 & \text{on } \partial\Omega \times (0,T) \\ u(.,0) = \omega & \text{in } \Omega \end{cases} \tag{1.1}$$

where Ω is a C^2 bounded domain in $\mathbb{R}^N$, $T > 0$, $g(u) \sim e^{a|u|^q}$ with $a > 0$ and $q \geq 1$, ω and μ are respectively bounded measures on Ω and Q_T.

In literature, the problem of existence and uniqueness for elliptic and parabolic equations involving exponential nonlinearity and measure data has been investigated by numerous authors. In [1], D. Bartolucci et al. proved that when $N > 2$, if ν is a bounded Radon measure on a bounded domain $\Omega \subset \mathbb{R}^N$ such that (C) $\nu \leq 4\pi\mathcal{H}^{N-2}$ (here $\mathcal{H}^{N-2}$ is $(N-2)$-dimensional Hausdorff measure in $\mathbb{R}^N$) then the problem

$$-\Delta u + e^u - 1 = \nu \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega \tag{1.2}$$

admits a unique solution. It was pointed out by A.C. Ponce that the converse is not true. However, when $N = 2$, J.L. Vazquez [11] showed that (C) is a necessary and sufficient condition for solving (1.2). Existence result for boundary value problem with measure data related to (1.2) was given by L. Véron [12]. Recently, a striking existence result for quasilinear elliptic equations was obtained by M.F. Bidaut-Véron et al. thanks to effective tool *Wolff potentials* (see [2] for more details).

Study on Cauchy problem for semilinear heat equations with exponential nonlinearity was carried out by many authors in different directions. See [3,4,9,10] and references therein. Among them, we refer to an interesting result in the framework of Orliz spaces due to B. Ruf and E. Terraneo [9]. They showed that local existence for the problem

$$\partial_t u - \Delta u - g(u) = 0 \quad \text{in } \mathbb{R}^N \times (0,T), \qquad u = u_0 \quad \text{in } \mathbb{R}^N \tag{1.3}$$

with $g(u) = e^{u^2} - 1$ can be established under a smallness condition on an appropriate Orliz norm of the initial data u_0. In [4], by using a contraction mapping argument, N. Ioku proved global existence for (1.3) under the same assumption on u_0. It is noteworthy that the method used in [9]