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Inflow problem for the one-dimensional compressible Navier–Stokes equations under large initial perturbation

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Abstract

This paper is concerned with the inflow problem for the one-dimensional compressible Navier–Stokes equations. For such a problem, Matsumura and Nishihara showed in [10] that there exists boundary layer solution to the inflow problem, and that both the boundary layer solution, the rarefaction wave, and the superposition of boundary layer solution and rarefaction wave are nonlinear stable under small initial perturbation. The main purpose of this paper is to show that similar stability results for the boundary layer solution and the supersonic rarefaction wave still hold for a class of large initial perturbation which can allow the initial density to have large oscillation. The proofs are given by an elementary energy method and the key point is to deduce the desired lower and upper bounds on the density function.

Keywords: Compressible Navier–Stokes equations; Boundary layer solution; Inflow problem; Rarefaction wave; Large initial perturbation; Large density oscillation

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1. Introduction

This paper is concerned with the large time behaviors of solutions to the inflow problem for one-dimensional compressible Navier–Stokes equations on the half line $\mathbb{R}_+ = (0, +\infty)$, which is an initial–boundary value problem in Eulerian coordinates:

$$\begin{cases} \rho_t + (\rho u)_x = 0, & \text{in } \mathbb{R}_+ \times \mathbb{R}_+, \\ (\rho u)_t + \big(\rho u^2 + \tilde{p}\big)_x = \mu u_{xx}, & \text{in } \mathbb{R}_+ \times \mathbb{R}_+, \\ (\rho, u)|_{x=0} = (\rho_-, u_-), & u_- > 0, \\ (\rho, u)(0, x) = (\rho_0, u_0)(x) \to (\rho_+, u_+), & \text{as } x \to +\infty. \end{cases} \tag{1.1}$$

Here, $\rho(>0)$, u, and $\tilde{p} = \tilde{p}(\rho) = \rho^\gamma$ with $\gamma \geq 1$ being the adiabatic exponent are, respectively, the density, the velocity, and the pressure, while the viscosity coefficient μ (>0), farfield states $\rho_\pm$ (>0) and $u_\pm$ are constants.

We assume that the initial data $(\rho_0(x), u_0(x))$ satisfy the boundary condition $(1.1)_3$ as a compatibility condition, i.e.

$$\rho_0(0) = \rho_-, \qquad u_0(0) = u_-.$$

The assumption $u_- > 0$ implies that, through the boundary $x = 0$ the fluid with the density ρ_- flows into the region $\mathbb{R}_+$, and hence the problem (1.1) is called the inflow problem. The cases of $u_- = 0$ and $u_- < 0$, the problems where the condition $\rho(t, 0) = \rho_-$ is removed, are called the impermeable wall problem and the outflow problem, respectively.

For the case of $u_- > 0$, as in [10], the inflow problem (1.1) can then be transformed to the problem in the Lagrangian coordinates:

$$\begin{cases} v_t - u_x = 0, & x > s_- t,\ t > 0, \\ u_t + p(v)_x = \mu \left(\dfrac{u_x}{v} \right)_x, & x > s_- t,\ t > 0, \\ (v, u)|_{x=s_- t} = (v_-, u_-), & u_- > 0, \\ (v, u)|_{t=0} = (v_0, u_0)(x) \to (v_+, u_+), & \text{as } x \to +\infty, \end{cases} \tag{1.2}$$

where