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Hadamard well-posedness for a hyperbolic equation of viscoelasticity with supercritical sources and damping

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Abstract

Presented here is a study of a viscoelastic wave equation with supercritical source and damping terms. We employ the theory of monotone operators and nonlinear semigroups, combined with energy methods to establish the existence of a unique local weak solution. In addition, it is shown that the solution depends continuously on the initial data and is global provided the damping dominates the source in an appropriate sense.

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Keywords: Viscoelastic; Memory; Integro-differential; Damping and sources; Monotone operators; Nonlinear semigroup

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1. Introduction

1.1. Literature overview

The theory of viscoelasticity encompasses description of materials that exhibit a combination of elastic (able to recover the original shape after stress application) and viscous (deformation-preserving after stress removal) characteristics. Quantitative description of such substances involves a strain–stress relation that depends on time. The classical linearized model yields an integro-differential equation that augments the associated elastic stress tensor with an appropriate *memory term* which encodes the history of the deformation gradient. The foundations of the theory go back to pioneering works of Boltzmann [10]. For fundamental modeling developments see [17] and the references therein.

When considering propagation of sound waves in viscoelastic fluids, if we neglect shear stresses then the stress tensor field may be expressed in terms of the acoustic pressure alone [31]. Thereby we obtain a scalar wave equation with a memory integral. This simplified formulation in fact captures most of the critical difficulties associated with the well-posedness of the viscoelastic vectorial model [17,30], and therefore the multi-dimensional scalar wave equation with memory will be the subject of the subsequent discussion.

One can consider such an integro-differential equation with a finite or infinite time delay (the former being a special case of the infinite-delay where the strain is zero for all $t < 0$). When restricted to the finite memory setting the system does not generate a semigroup, whereas the infinite-delay model can be represented as a semigroup evolution with the help of an appropriately defined history variable.

The (linear) viscoelastic problem with infinite memory and its stability were extensively addressed in [18,19,21]. Existence of global attractors for wave equations with infinite memory in the presence of nonlinear sources and linear internal damping (velocity feedback) was investigated in [30]. The "source" here refers to amplitude-dependent feedback nonlinearity whose growth rate is polynomially bounded with exponent $p \geq 1$. Due to the regularity of finite-energy solutions for this problem—H^1 Sobolev level for the displacement variable—the source considered in the latter reference was *subcritical* ($p < n/(n-2)$ for dimensions $n > 2$) with respect to this energy. Subsequently in [20] the authors look at attractors for the problem with strong (Kelvin–Voigt) damping and higher-order sources, including exponents of maximal order for which the associated energy is defined ($p = 5$ in 3D).

A larger body of work is available on the finite-time delay problem. The papers in this list focus predominantly on well-posedness and asymptotic stability with energy dissipation due to memory and interior and/or boundary velocity feedbacks. In addition, the sources, if present are at most critical, i.e., $p \leq n/(n-2)$ in dimensions above 2. See [13] for a treatment of interior and boundary memory with nonlinear boundary damping and no sources. Energy decay was investigated under localized interior dissipation and a source term was addressed in [14,15]. Local and global well-posedness with source, but now without additional frictional damping was the subject of the paper [3]. For systems of coupled waves with memory see [26]. Recent blow-up results for viscoelastic wave equations can be found in [27,28]. For quasilinear viscoelastic models with no sources and Kelvin–Voigt damping refer for example to [12,29].

Overall, it appears that the finite-time memory case has been more actively studied. Yet to our knowledge presently there are no works dealing with super-critical source exponents (i.e., $p > 3$ in 3D) in combination with memory terms. In light of this trend the present goal of this paper is two-fold: