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Normal forms for semilinear equations with non-dense domain with applications to age structured models

Zhihua Liu [a,1], Pierre Magal [b,c,d,∗,2], Shigui Ruan [e,3]

[a] *School of Mathematical Sciences, Beijing Normal University, Beijing 100875, People's Republic of China*
[b] *Univ. Bordeaux, IMB, UMR 5251, F-33400 Talence, France*
[c] *CNRS, IMB, UMR 5251, F-33400 Talence, France*
[d] *School of Mathematics, Sichuan University, Chengdu 610064, People's Republic of China*
[e] *Department of Mathematics, University of Miami, Coral Gables, FL 33124-4250, USA*

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Abstract

Normal form theory is very important and useful in simplifying the forms of equations restricted on the center manifolds in studying nonlinear dynamical problems. In this paper, using the center manifold theorem associated with the integrated semigroup theory, we develop a normal form theory for semilinear Cauchy problems in which the linear operator is not densely defined and is not a Hille–Yosida operator and present procedures to compute the Taylor expansion and normal form of the reduced system restricted on the center manifold. We then apply the main results and computation procedures to determine the direction of the Hopf bifurcation and stability of the bifurcating periodic solutions in a structured evolutionary epidemiological model of influenza A drift and an age structured population model.

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Keywords: Normal form; Non-densely defined Cauchy problem; Center manifold; Hopf bifurcation; Structured population; Population dynamics

* Corresponding author.
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1. Introduction

1.1. Normal form theory

To determine the qualitative behavior of a nonlinear system in the neighborhood of a nonhyperbolic equilibrium point, the center manifold theorem implies that it could be reduced to the problem of determining the qualitative behavior of the nonlinear system restricted on the center manifold, which reduces the dimension of a local bifurcation problem near the nonhyperbolic equilibrium point. The normal form theory provides a way of finding a nonlinear analytic transformation of coordinates in which the nonlinear system restricted on the center manifold takes the "simplest" form, called normal form. These two methods, one reduces the dimension of the original system and the other eliminates the nonlinearity of the reduced system, are conjunctly used to study bifurcations in nonlinear dynamical systems. A normal form theorem was obtained first by Poincaré [54] and later by Siegel [56] for analytic differential equations. Simpler proofs of Poincaré's theorem and Siegel's theorem were given in Arnold [5], Meyer [46], Moser [48], and Zehnder [67]. For more results about normal form theory and its applications see, for example, the monographs by Arnold [5], Chow and Hale [8], Guckenheimer and Holmes [26], Meyer and Hall [47], Siegel and Moser [57], Chow et al. [9], Kuznetsov [34], and others.

Normal form theory has been extended to various classes of partial differential equations. In the context of autonomous partial differential equations we refer to Ashwin and Mei [6] (PDEs on the square), Eckmann et al. [18] (abstract parabolic equations), Faou et al. [20,21] (Hamiltonian PDEs), Hassard, Kazarinoff and Wan [28] (functional differential equations), Faria [22,23] (PDEs with delay), Foias et al. [25] (Navier–Stokes equation), Kokubu [33] (reaction–diffusion