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Polynomial normal forms of constrained differential equations with three parameters

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Abstract

We study generic constrained differential equations (CDEs) with three parameters, thereby extending Takens's classification of singularities of such equations. In this approach, the singularities analyzed are the Swallowtail, the Hyperbolic, and the Elliptic Umbilics. We provide polynomial local normal forms of CDEs under topological equivalence. Generic CDEs are important in the study of slow–fast (SF) systems. Many properties and the characteristic behavior of the solutions of SF systems can be inferred from the corresponding CDE. Therefore, the results of this paper show a first approximation of the flow of generic SF systems with three slow variables.
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Keywords: Constrained differential equations; Slow–fast systems; Normal forms; Catastrophe theory

Contents

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1. Introduction

The present document studies *constrained differential equations* (CDEs) with three parameters. The main motivation comes from *slow–fast* systems, which are usually given as

$$\begin{aligned} \varepsilon\dot{x} &= f(x,\alpha,\varepsilon), \\ \dot{\alpha} &= g(x,\alpha,\varepsilon), \end{aligned} \tag{1.1}$$

where $x \in \mathbb{R}^n$ represents states of a process, $\alpha \in \mathbb{R}^m$ denotes control parameters, and $\varepsilon > 0$ is a small constant. Mathematical equations as (1.1) are often used to model phenomena with two time scales. A constrained differential equation is the limit $\varepsilon = 0$ of (1.1), that is

$$\begin{aligned} 0 &= f(x,\alpha,0), \\ \dot{\alpha} &= g(x,\alpha,0). \end{aligned} \tag{1.2}$$

We assume throughout the rest of the text that the functions $f(\cdot)$ and $g(\cdot)$ are $\mathcal{C}^\infty$ smooth (all partial derivatives exist and are continuous). From (1.1) one can observe that whenever $f(\cdot) \neq 0$, the smaller ε is, the faster x evolves with respect to α. Therefore, in the context of SF systems, the coordinates x and α receive the name of *fast* and *slow* respectively. Defining the new time parameter $\tau = t/\varepsilon$, the system (1.1) can be rewritten as

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