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Global asymptotic stability of constant equilibria in a fully parabolic chemotaxis system with strong logistic dampening

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Abstract

This paper deals with nonnegative solutions of the Neumann initial–boundary value problem for the parabolic chemotaxis system

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + u - \mu u^2, & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \end{cases}$$

in bounded convex domains $\Omega \subset \mathbb{R}^n$, $n \geq 1$, with smooth boundary.

It is shown that if the ratio $\frac{\mu}{\chi}$ is sufficiently large, then the unique nontrivial spatially homogeneous equilibrium given by $u = v \equiv \frac{1}{\mu}$ is globally asymptotically stable in the sense that for any choice of suitably regular nonnegative initial data (u_0, v_0) such that $u_0 \not\equiv 0$, the above problem possesses a uniquely determined global classical solution (u, v) with $(u, v)|_{t=0} = (u_0, v_0)$ which satisfies

$$\left\| u(\cdot,t) - \frac{1}{\mu} \right\|_{L^\infty(\Omega)} \to 0 \quad \text{and} \quad \left\| v(\cdot,t) - \frac{1}{\mu} \right\|_{L^\infty(\Omega)} \to 0$$

as $t \to \infty$.

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1. Introduction

1.1. Chemotaxis with logistic cell kinetics

In the understanding of collective behavior in cell populations in biology, the partially oriented movement of cells in response to chemical signals, aka *chemotaxis*, is known to play a crucial role in various contexts. This importance partly stems from the fact that when combined with the ability of cells to produce the respective signal substance themselves, chemotaxis mechanisms are among the most primitive forms of intercellular communication. Typical examples include aggregation processes such as slime mold formation in *Dictyostelium Discoideum* [15] or pattern formation like e.g. in colonies of *Salmonella typhimurium* [35], but also medically relevant processes such as tumor invasion [3,17] and self-organization during embryonic development [24]. For a broad overview over various types of chemotaxis processes, we refer the reader to the survey [11] and the references therein.

In numerous cases, the time scales of chemotactic migration interfere with those of cell proliferation and death. It is known that the interplay of these mechanisms may result in quite colorful dynamical behavior, and numerical simulations suggest that some of these facets can already be described mathematically by straightforward extensions of the classical Keller–Segel chemotaxis model [15] such as the parabolic system

$$\begin{cases} u_t = d_1\Delta u - \chi\nabla\cdot(u\nabla v) + ru - \mu u^2, & x\in\Omega,\ t>0,\\ \tau v_t = d_2\Delta v - \alpha v + u, & x\in\Omega,\ t>0, \end{cases} \tag{1.1}$$

for the cell density $u = u(x,t)$ and the signal concentration $v = v(x,t)$ in the physical domain $\Omega\subset\mathbb{R}^n$, under appropriate choices of the parameters d_1, d_2, χ, r, μ, α and τ [12]. Some corresponding rigorous analytical evidence for the occurrence of colorful solution behavior in such models has recently been gained in [34]: It has been shown there that in the Neumann problem for the spatially one-dimensional version of (1.1) with $\tau = 0$, $d_2 = \alpha = \chi = 1$, under a smallness assumption on d_1 the solution component u may exceed the respective carrying capacity $\frac{r}{\mu}$ to an arbitrary extent at some intermediate time. After all, the logistic cell kinetic term in (1.1) exerts a somewhat stabilizing influence on the system in the sense of blow-up prevention: Whereas in the case $r = \mu = 0$ corresponding to the classical Keller–Segel system, solutions may become unbounded within finite time when $n\ge 2$ [10,20,33,18], it is known that arbitrarily small $\mu > 0$ enforce global existence and boundedness of solutions when $n\le 2$, and that suitably large μ similarly rule out blow-up in the case $n\ge 3$ [22,23,29,32].

1.2. Main result

It is the goal of the present paper to investigate in more detail how the destabilizing and aggregation-supporting properties of chemotactic cross-diffusion interact with growth limitations of logistic type. Having in mind this focusing, for simplicity in presentation we conveniently normalize all parameters in (1.1) except for χ and μ and thus henceforth specifically consider the prototypical parabolic initial–boundary value problem

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + u - \mu u^2, & x\in\Omega,\ t>0,\\ v_t = \Delta v - v + u, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), & x\in\Omega, \end{cases} \tag{1.2}$$