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Time periodic traveling wave solutions for periodic advection–reaction–diffusion systems

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Abstract

We study the existence, uniqueness, and asymptotic stability of time periodic traveling wave solutions to a class of periodic advection–reaction–diffusion systems. Under certain conditions, we prove that there exists a maximal wave speed c^* such that for each wave speed $c \leq c^*$, there is a time periodic traveling wave connecting two periodic solutions of the corresponding kinetic system. It is shown that such a traveling wave is unique modulo translation and is monotone with respect to its co-moving frame coordinate. We also show that the traveling wave solutions with wave speed $c \leq c^*$ are asymptotically stable in certain sense. In addition, we establish the nonexistence of time periodic traveling waves with speed $c > c^*$.

MSC: 35B10; 35B35; 35B40; 35C07; 35K40

Keywords: Advection–reaction–diffusion system; Time periodic traveling waves; Asymptotic stability; Maximal wave speed; Lotka–Volterra competition system

Contents

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1. Introduction

Traveling wave solutions of reaction–diffusion systems have been studied intensively over the last four decades since wave phenomena are observed in many time dependent processes described by evolution equations (see Conley and Gardner [8], Dunbar [9], Gardner [12], Gourley and Ruan [15], Hosono [19], Kan-On [20], Lewis et al. [21], Li et al. [22], Sandstede and Scheel [31], Volpert et al. [32], Weinberger [34] and references therein). Moreover, the study of traveling wave solutions has been such an essential part of mathematical analysis of evolving spatial patterns generated by nonlinear parabolic equations because of their importance in governing the long time behavior and stability.

Although the study of traveling wave solutions has a longstanding history, most of the existing studies are devoted to autonomous equations. Recently, an interest in both space and time periodic traveling wave solutions has been stimulated by a vast number of examples of biological and physical systems where relevant parameters are either space periodic (Berestycki and Hamel [4], Berestycki et al. [5–7]) or time periodic (Alikakos et al. [1], Liang et al. [24], Liang and Zhao [25], Nolen and Xin [30], Xin [35], Zhao [36]). For pulsating fronts, Hamel [16] and Hamel and Roques [17] presented a systematic analysis of the qualitative behavior, uniqueness, and stability of monostable pulsating fronts for reaction–diffusion equations in periodic media with KPP nonlinearities. The established results provide a complete classification of all KPP pulsating fronts. Most recently, Zhao and Ruan [37] investigated time periodic traveling wave solutions of a diffusive Lotka–Volterra competition with periodic forcing. The basic existence and uniqueness results for traveling waves connecting two semi-trivial periodic solutions of the corresponding kinetic system were obtained. The asymptotic stability of traveling wave solutions was also established.

On the other hand, advection–reaction–diffusion equations have been used extensively to model some reaction–diffusion processes taking place in moving media such as fluids, for example, combustion, atmospheric chemistry, and plankton distributions in the sea, etc. Berestycki [2], Gilding and Kersner [14], Malaguti and Marcelli [28], and Malaguti et al. [29] investigated the influence of advection on the propagation of traveling wave fronts in some reaction–diffusion systems. See also Liang and Wu [23] and Wang et al. [33].

In this paper, we are interested in studying the existence and other qualitative behaviors of time periodic traveling wave solutions of a periodic advection–reaction–diffusion system of the following form:

$$\begin{cases} u_t = d_1(t)\Delta u + \mathbf{k}(t)\cdot\nabla u + f(t,u,v), \\ v_t = d_2(t)\Delta v + \mathbf{l}(t)\cdot\nabla v + g(t,u,v), \end{cases} \tag{1.1}$$

where $u = u(t,x)$, $v = v(t,x)$, $(t,x) \in \mathbb{R}^+ \times \mathbb{R}^n$ $(n \geq 1)$, $\Delta := \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$, $\nabla := (\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_n})$, $\mathbf{k}(t) = (k_1(t), \cdots, k_n(t))$, $\mathbf{l}(t) = (l_1(t), \cdots, l_n(t))$, d_i $(i = 1,2)$ and k_i and l_i $(i = 1, \cdots, n)$ are T-periodic and Hölder continuous functions of t, d_i is strictly positive in $[0,T]$, while k_i and l_i