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A global unique solvability of entropic weak solution to the one-dimensional pressureless Euler system with a flocking dissipation ☆

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Abstract

We present a unique global solvability and flocking estimate of an entropic weak solution to the one-dimensional pressureless Euler system with a flocking dissipation in all-to-all coupling setting. This model appears naturally as a quasi-equilibrium model for hydrodynamic description of Cucker–Smale flocking. For the unique global solvability, we adopt the variation approach from Ding and Huang's work [19] on the inhomogeneous pressureless gas dynamic model. When initial mass and velocity are locally integrable and bounded measurable functions, respectively, we give explicit representations for the global entropic weak solutions. Our results do not require any smallness of initial data except that initial mass density is almost everywhere positive.

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1. Introduction

Collective coherent motions of self-propelled agents (particles), e.g., flocking of birds, schooling of fish, swarming of bacteria and herding of cows, etc. are often found in our biological systems [30,35,41]. Recently, research on the collective coherent motions has received considerable attention from many scientific disciplines such as applied math, biology, computer science, statistical physics and control theory due to their diverse applications [22,31,36–38,40] in relation with the decentralized control of multi-agents such as UAVs and robots. The jargon "flocking" represents a phenomenon in which self-propelled agents are organized into the ordered motion using only limited environmental information and simple rules. In this paper, we are particularly interested in the velocity flocking, in which all agents move with the same velocity, but their spatial positions are distributed. So far, several phenomenological flocking models have been proposed in literature. Among them, our main interest lies on the simple ODE system proposed by Cucker and Smale [16].

Consider infinitely many Cucker–Smale (for short C–S) agents whose phase configuration is close to a flocking one. In this case, the dynamics of C–S agents can be effectively approximated by a quasi-equilibrium hydrodynamic model. Let ρ and u be the local mass and velocity densities of C–S agents exhibiting the flocking phenomenon, respectively. Then, the temporal-spatial evolution of (ρ, u) is governed by the Cauchy problem to the quasi-equilibrium C–S hydrodynamic model:

$$\begin{cases} \partial_t \rho + \partial_x(\rho u) = 0, \quad x \in \mathbb{R},\ t > 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) = -K\rho \displaystyle\int_{\mathbb{R}} \psi\big(|x-y|\big)\big(u(x,t) - u(y,t)\big)\rho(y,t)dy, \\ (\rho, u)(x,0) = (\rho_0, u_0), \end{cases} \tag{1.1}$$

where K is the positive coupling strength and $\psi(|x-y|)$ is a Lipschitz continuous communication weight satisfying

$$\psi \geq 0, \quad \|\psi\|_{L^\infty} + \|\psi\|_{\mathrm{Lip}} < \infty, \qquad \big(\psi(r_1) - \psi(r_2)\big)(r_1 - r_2) \leq 0, \quad r_1, r_2 > 0.$$

The system (1.1) appears as an approximate model for the hydrodynamic Cucker–Smale model [27], when the continuum flocking group is close to a flocking configuration (see Section 2.1). Note that in the zero coupling limit $K \to 0$, the system (1.1) reduces to the pressureless Euler system, which has been extensively studied in the hyperbolic conservation law community in last decades. For a detailed discussion on literature, we refer to Section 2.2.

The purpose of this paper is to address a unique solvability and flocking estimate of entropic weak solution to the system (1.1) in one-dimensional and all-to-all coupling setting ($\psi = 1$) using the variational arguments of [19,20,42–44]. In this all-to-all coupling case, the system (1.1) can be reduced to the pressureless Euler system with a damping under some suitable normalization. So we might have some possibility to apply the variational method to construct entropic weak solutions as in [19] for the pressureless Euler system with some specific source term.