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Recovering complex elastic scatterers by a single far-field pattern

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Abstract

We consider the inverse scattering problem of reconstructing multiple impenetrable bodies embedded in an unbounded, homogeneous and isotropic elastic medium. The inverse problem is nonlinear and ill-posed. Our study is conducted in an extremely general and practical setting: the number of scatterers is unknown in advance; and each scatterer could be either a rigid body or a cavity which is not required to be known in advance; and moreover there might be components of multiscale sizes presented simultaneously. We develop several locating schemes by making use of only a single far-field pattern, which is widely known to be challenging in the literature. The inverse scattering schemes are of a totally "direct" nature without any inversion involved. For the recovery of multiple small scatterers, the nonlinear inverse problem is linearized and to that end, we derive sharp asymptotic expansion of the elastic far-field pattern in terms of the relative size of the cavities. The asymptotic expansion is based on the boundary-layer-potential technique and the result obtained is of significant mathematical interest for its own sake. The recovery of regular-size/extended scatterers is based on projecting the measured far-field pattern into an admissible solution space. With a local tuning technique, we can further recover multiple multiscale elastic scatterers.
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1. Introduction

This work concerns the time-harmonic elastic scattering from cavities (e.g., empty or fluid-filled cracks and inclusions) and rigid bodies, which has its origin in industrial and engineering applications; see, e.g., [30,9,21,22] and the references therein. In seismology and geophysics, it is important to understand how anomalies diffract the detecting elastic waves and to characterize them from the surface measurement data. This leads to the inverse problem of determining the position and shape of an elastic scatterer; see, e.g., [1,12,13]. The inverse elastic scattering problem also plays a key role in many other science and technology such as petroleum and mine exploration, nondestructive testing of concrete structures etc. The inverse problem is nonlinear and ill-posed and far from well understood. In this work, we shall develop several qualitative inverse elastic scattering schemes in an extremely general and practical scenario. In what follows, we first present the mathematical formulations of the forward and inverse elastic scattering problems for the present study, and then we briefly discuss the results obtained.

Consider a time-harmonic elastic plane wave $u^{in}(x)$, $x \in \mathbb{R}^3$ (with the time variation of the form $e^{-i\omega t}$ being factorized out, where $\omega \in \mathbb{R}_+$ denotes the frequency) impinging on a scatterer $D \subset \mathbb{R}^3$ embedded in an infinite isotropic and homogeneous elastic medium in $\mathbb{R}^3$. The incident elastic plane wave is of the following general form

$$u^{in}(x) = u^{in}\big(x; d, d^{\perp}, \alpha, \beta, \omega\big) = \alpha d e^{ik_p x\cdot d} + \beta d^{\perp} e^{ik_s x\cdot d}, \quad \alpha, \beta \in \mathbb{C}, \tag{1.1}$$

where $d \in \mathbb{S}^2 := \{x \in \mathbb{R}^3 : |x| = 1\}$, is the impinging direction, $d^{\perp} \in \mathbb{S}^2$ satisfying $d^{\perp} \cdot d = 0$ denotes the polarization direction; and $k_s := \omega/\sqrt{\mu}$, $k_p := \omega/\sqrt{\lambda + 2\mu}$ denote the shear and compressional wave numbers, respectively. If $\alpha = 1, \beta = 0$ for u^{in} in (1.1), then $u^{in} = u_p^{in} := de^{ik_p x\cdot d}$ is the (normalized) plane pressure wave; and if $\alpha = 0, \beta = 1$ for u^{in} in (1.1), then $u^{in} = u_s^{in} := d^{\perp} e^{ik_s x\cdot d}$ is the (normalized) plane shear wave. Let $u(x) \in \mathbb{C}^3$, $x \in \mathbb{R}^3 \backslash \overline{D}$ denote the total displacement field, and define the linearized strain tensor by

$$\varepsilon(u) := \frac{1}{2}\big(\nabla u + \nabla u^{\top}\big) \in \mathbb{C}^{3\times 3}, \tag{1.2}$$

where ∇u and $\nabla u^{\top}$ stand for the Jacobian matrix of u and its adjoint, respectively. By Hooke's law the strain tensor is related to the stress tensor via the identity

$$\sigma(u) = \lambda(\operatorname{div} u)\mathbf{I} + 2\mu\varepsilon(u) \in \mathbb{C}^{3\times 3} \tag{1.3}$$

with the Lamé constants λ, μ satisfying $\mu > 0$ and $3\lambda + 2\mu > 0$. Here and in what follows, $\mathbf{I}$ denotes the 3×3 identity matrix. The surface traction (or the stress operator) on ∂D is defined as

$$Tu = T_\nu u := \nu \cdot \sigma(u) = (2\mu\nu \cdot \operatorname{grad} + \lambda\nu \operatorname{div} + \mu\nu \times \operatorname{curl})u, \tag{1.4}$$

where ν denotes the unit normal vector to ∂D pointing into $\mathbb{R}^3 \backslash \overline{D}$. We suppose that $D \subset \mathbb{R}^3$ is a bounded C^2 domain such that $\mathbb{R}^3 \backslash \overline{D}$ is connected. For the subsequent use, we also introduce $Ru := u$. In the present study, the elastic body D is supposed to be either a cavity or a rigid body for which u satisfies the following boundary condition

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