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Regularity results for a new class of functionals with non-standard growth conditions

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We prove that local minimizers of the functional

$$\mathcal{F}(u) := \int_{\Omega} |Du|^{p(x)} \log\bigl(e + |Du|\bigr)\,dx$$

are of class $C^{0,\alpha}$ for every $0 < \alpha < 1$, if the exponent $p(x) > 1$ has logarithmic modulus of continuity. Moreover, in case the exponent $p(x) > 1$ is a Hölder continuous function, we establish that minimizers of $\mathcal{F}(u)$ are of class $C^{1,\alpha}$, for some $0 < \alpha < 1$.

1. Introduction

The aim of this paper is the study of the regularity properties of local minimizers of integral functionals of the type

$$\int_{\Omega} f(x, Du)\,dx,$$

where Ω is a bounded open set in $\mathbb{R}^n$, $f : \Omega \times \mathbb{R}^{nN} \to \mathbb{R}$ is a Carathéodory function and u belongs to the Sobolev class $W^{1,1}_{\mathrm{loc}}(\Omega; \mathbb{R}^N)$. We recall that, under the assumption of p-growth on the integrand f, i.e.

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$$|z|^p \leqslant f(x,z) \leqslant L\big(1+|z|^p\big), \quad p>1, \tag{1.1}$$

there is a wide literature on the regularity theory under natural convexity or quasiconvexity assumption on the integrand f (for an exhaustive treatment of this subject we refer to [22,23] and the references therein).

In 1991, P. Marcellini started the study of the regularity of local minimizers under the more flexible (p,q)-growth assumption

$$|z|^p \leqslant f(x,z) \leqslant L\big(1+|z|^q\big), \quad 1<p<q \tag{1.2}$$

(see [27–29]), stimulating the interest for the theory of regularity of integrals with non-standard growth conditions and that, since then, have received many contributions from several authors (see for example [1,5,8,9,14–16,30]). An interesting class of functionals that satisfy non-standard growth conditions are those with φ-growth, i.e.

$$\varphi\big(|z|\big) \leqslant f(x,z) \leqslant L\big(1+\varphi\big(|z|\big)\big), \tag{1.3}$$

where φ is a given Orlicz function. Regularity of local minimizers of functionals with φ-growth, under suitable assumption on the function φ, has been studied for example in [4,12,19–21].

A borderline case lying between (1.1) and (1.2) is the one of $p(x)$-growth

$$|z|^{p(x)} \leqslant f(x,z) \leqslant L\big(1+|z|^{p(x)}\big), \quad p(x)>1. \tag{1.4}$$

In the last few years, the regularity theory for functionals whose integrand satisfies $p(x)$-growth assumptions has been dealt with in many papers also in view of their connection with problems emerging from mathematical physics.

The starting point for the regularity theory of problems with $p(x)$-growth is the paper [32] by Zhikov who proved the higher integrability of local minimizers of the model functional

$$\int_\Omega |Du|^{p(x)}\,dx, \tag{1.5}$$

provided the exponent $p(x)$ has logarithmic modulus of continuity, i.e. if there exists a function $\omega:\mathbb{R}^+\to\mathbb{R}^+$, nondecreasing, continuous and vanishing at zero such that

$$\big|p(x)-p(y)\big| \leqslant \omega\big(|x-y|\big), \qquad \limsup_{R\to 0}\omega(R)\log\frac{1}{R}<+\infty. \tag{1.6}$$

After this initial regularity result, localization and "freezing" techniques have been successfully employed to obtain higher regularity such as $C^{0,\alpha}$ or even $C^{1,\alpha}$ under suitable regularity assumption for $p(x)$, see [2,7]. These results were followed by a variety of contributions, see for example [3,13,17,24].

As Zhikov himself proved in [32], it is worth pointing out that condition (1.6) is sharp in the sense that dropping it causes the loss of any type of regularity, even of higher integrability. Moreover functionals with $p(x)$-growth exhibit the so-called Laurentiev phenomenon if and only if (1.6) is violated.

In this paper, we will study the regularity properties of the minimizers of the functional

$$\mathcal{F}(u) := \int_\Omega |Du|^{p(x)}\log\big(e+|Du|\big)\,dx, \tag{1.7}$$