



# On the controllability of the non-isentropic 1-D Euler equation

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## Abstract

In this paper, we examine the question of the boundary controllability of the one-dimensional non-isentropic Euler equation for compressible polytropic gas, in the context of weak entropy solutions. We consider the system in Eulerian coordinates and the one in Lagrangian coordinates. We obtain for both systems a result of controllability toward constant states (with the limitation  $\gamma < \frac{5}{3}$  on the adiabatic constant for the Lagrangian system). The solutions that we obtain remain of small total variation in space if the initial condition is itself of sufficiently small total variation.

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*Keywords:* Boundary control; Controllability; Hyperbolic systems of conservation laws; Entropy solutions; Compressible fluids

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